AG-522 Seat No.________

B. Sc. (Sem. IV) Examination

March - 2019

CC - MATH - 402 : Mathematics

(Advanced Linear Algebra)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) This question paper contains four questions and all questions are compulsory.

(2) Figures to the right side indicates the marks of the corresponding question.

1 (a) A square matrix A is invertible if and only if the corresponding linear transformation T is non-singular. **8**

OR

(a) If A is $m \times n$ matrix and B is $n \times p$ matrix then prove that $(AB)^T = B^T A^T$ **8**

(b) Attempt any **two** : **12**

(1) Solve the system of equations :

$$x + y + z = 6$$
$$2x - y + z = 3$$
$$x + 4y - z = 6$$

by row reduction method.

(2) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and

$B_1 = \{(1,1,1),(1,0,0),(0,1,0)\}$

$B_2 = \{(1,2,3),(1,-1,1),(2,1,1)\}$

are ordered basis of vactor space R^3 then find a linear transformation $T : R^3 \to R^3$ such that $A = [T : B_1, B_2]$.

(3) Determine the rank of matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ by row reduction echelon form.

2 (a) State and prove Schwartz's inequality. 8

OR

(a) Let V be an n-dimensional vector space and $B = \{x_1, x_2, x_3, \ldots\ldots, x_n\}$ be a basis of V then there exist Uniquely basis $B^* = \{f_1, f_2, f_3, \ldots\ldots f_n\}$ of V^* such that $f_i(x_j) = \delta_{ij}$ $i, j = 1, 2, 3 \ldots n$. 8

(b) Attempt any **two** : 12

(i) In a vector space R^2 for $x = (x_1, x_2)$ $y = (y_1, y_2) \in R^2$ define $\langle x, y \rangle = x_1 y_1 - 2x_1 y_2 + 3x_2 y_1 + x_2 y_2$ then show that $\langle x, y \rangle$ is an inner product on vector space R^2.

(ii) Using Gram-Schmidt process obtain the orthonormal basis from the basis $\{(1,0,1),(1,0,-1),(0,3,4)\}$.

(iii) Define a dual basis. Find dual basis for the base $\{(1,-1,3),(0,-1,-1),(0,3,-2)\}$

3 (a) Staste and prove Cayley-Hamilton theorem. **8**

OR

(a) Prove that eigen vectors of symmetric linear map corresponding to different eigen values are perpendicular to each other. **8**

(b) Attempt any **two** : **12**

(i) Find eigen values and corresponding eigen vectors of matrix.

$$A=\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$$

(ii) Usindg Cayley-Hamilton theorem find the inverse of matrix

$$A=\begin{bmatrix}1 & 3 & 2\\ 0 & 2 & 1\\ 0 & 5 & 3\end{bmatrix}$$

(iii) Find the minimal polynomial for matrix

$$A=\begin{bmatrix}2 & 3\\ 1 & 5\end{bmatrix}$$

4 Attempt any **five** : **10**

(i) Prove that $(AB)^{-1} = B^{-1}A^{-1}$

(ii) Define Rank and Nullity of a matrix.

(iii) State and prove 'Parallelogram Law'.

(iv) Define : Inner Product Space.

(v) Define : Symmetric transformation, Eigen value of matrix.

(vi) Define : Normal operator, unitary operator.

(vii) For linear mapping : $R^2 \to R^2, T(x, y) = (x + 2y, x - y), \forall x, y \in R$ find $T^*(3, 2)$
