AG-520

Seat No._____

B. Sc. (Sem. IV) Examination

March - 2019

Mathematics : CC MATH - 401

(Advanced Calculus)

Time : 3 Hours] [Total Marks : **70**

Instructions :

(1) All questions are compulsory.

(2) Figure to the right indicates the marks of the corresponding question.

1 (a) Obtain the formula for radius of curvature of the curve $y = f(x)$. **8**

OR

(a) Prove that $\beta(m,n) = \dfrac{\lceil m \lceil n}{\lceil m+n}$.

(b) Attempt any **three** : **12**

(1) Find the radius of curvature of $r^n = a^n \sin n\theta$.

(2) Find the double point of the curve $x^3 + y^3 - 3axy = 0$ and explain their type.

(3) Prove that $\int_0^1 \sqrt{x}\sqrt[3]{1-x^2}\,dx = \frac{1}{2}\beta\left(\frac{3}{4},\frac{4}{3}\right)$

(4) Prove that $\int_0^1 x^{q-1}\left(\log,\frac{1}{x}\right)^{p-1} dx = \dfrac{\lceil p}{q^p}$.

2 (a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \left(x^2+y^2\right) dx\, dy$ by transforming into polar co-ordinate. 8

OR

(a) Evaluate : $\iiint_V xyz\, dxdydz$, where

$V = \left\{(x, y, z) / x, y, z \geq 0, x^2 + y^2 + z^2 \leq 1\right\}$

(b) Attempt any **three** : 12

(1) Change the order of integration

$$\int_0^3 \int_{4y/3}^{\sqrt{25-y^2}} f(x, y)\, dy\, dx$$

(2) Evaluate : $\iint xy(x^2 + y^2)\, dx.dy$, where

$s = [0, a] \times [0, b]$

(3) Find the volume of a sphere $x^2 + y^2 + z^2 = a^2$.

(4) Evaluate : $\int_0^a \int_0^{\sqrt{a^2-b^2}} x^2 y\, dx\, dy,\ x \geq 0,\ y \geq 0.$

3 (a) State and prove Green's Theorem. 8

OR

(a) Prove that $curl(\varphi f) = \varphi\, curl\, f + (grad\, \varphi) \times f$.

(b) Attempt any **two** : **12**

(1) If $f = \left(2yz, -x^2y, xz^2\right)$ and

$\phi(x, y, z) = 2x^2yz^3$ then $(f.\nabla)\,\phi = f \cdot (\nabla\phi)$.

(2) Find div F and Curl F, where

$F = \text{grad}\left(x^3 + y^3 + z^3 - 3xyz\right)$.

(3) Evaluate $\iint_S f.n\,ds$, where

$f = \left(x + y^2, -2x, 2yz\right)$ and surface s is the plane. $2x + y + 2z = 6$ in the first octant.

4 Attempt any **three** : **10**

(1) Find the radius of curvature of $x = 3\cos\theta,\ y = 2\sin\theta$ at $\theta = \pi/2$.

(2) Explain type of double point (2, 3) of the curve $x^3 + y^3 - 12x - 27y + 70 = 0$.

(3) Evaluate : $\int_0^\infty x^4 e^{-x^4}\,dx$.

(4) If $\phi(x, y, z) = 3x^2y^2z + 4xy^2z^2$ then find *grad* ϕ at point (1,1,1).

(5) Evaluate : $\int_0^1 \int_0^3 \left(x^2 + y^2\right) dx\,dy$.
