ABC-2967 Seat No.________

B. Sc. (Sem. II) Examination

March / April – 2019

Mathematics : CCMATH-122

Time : **3** Hours] [Total Marks : **70**

1 (a) State and prove De-Moiver's theorem. **7**

OR

Prove that

$$\cos n\theta = \cos^n \theta - \binom{n}{2}\cos^{n-2}\theta \cdot \sin^2\theta + \binom{n}{4}\cos^{n-4}\theta \cdot \sin^4\theta - \ldots\ldots$$

where $n \in N$.

(b) Attempt any two : **8**

(1) If $x^2 - 2x\cos\theta + 1 = 0$ then prove that, $x^{2n} - 2x^n \cos n\theta + 1 = 0$.

(2) Prove that :

$$\sin^7\theta = \frac{1}{64}\left[35\sin\theta - 21\sin 3\theta + 7\sin 5\theta - \sin 7\theta\right].$$

(3) Construct the equation whose roots are $2\cos\frac{\pi}{7}, 2\cos\frac{3\pi}{7}$ and $2\cos\frac{5\pi}{7}$.

2 (a) Prove that : $\cosh^{-1}(z) = \log\left(z + \sqrt{z^2 - 1}\right)$. 7

OR

Show that : $\sum \frac{n^p}{\sqrt{n+1} - \sqrt{n}}$ is convergent if $p < -\frac{1}{2}$ and it is divergent if $p \geq -\frac{1}{2}$.

(b) Attempt any two : 8

(1) If $\cosh(x) = \sec\theta$ then prove that $\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = e^x$.

(2) If $i^{\alpha + i\beta} = \alpha + i\beta$ then prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\beta\pi}$.

(3) Discuss the convergence of the following series :

$$\sum\left(1 - \frac{1}{n}\right)^{n^2} \text{ and } \sum\left(\sqrt[n]{n} - \frac{1}{2}\right)^n x^n.$$

3 (a) Explain the general solution of Bernoulli's differential equation. 7

OR

Prove that $\frac{1}{f(D)}\left(e^{ax} \cdot V\right) = e^{ax} \cdot \frac{1}{f(D+a)} V$. Where V is a function of x.

(b) Solve any two : **8**

(1) $y\sqrt{1+p^2} = (x+yp)^2$

(2) $D^3\left(D^2-1\right)y = 5x^2$

(3) $y''-5y'+6y = x^2e^x$

4 (a) If $A = \left[a_{ij}\right]_{n\times n}$ then prove that, **7**

$A(adj\ A) = (adj\ A)A = |A|I_n$.

OR

If A and B are matrices of order m×n and n×p respectively then, prove that $(AB)^T = B^T A^T$.

(b) Attempt any two : **8**

(1) Find inverse of a matrix $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ by Row-Reduction method.

(2) Obtain the rank of matrix

$$B = \begin{bmatrix} 3 & 0 & 0 & 2 \\ 1 & 2 & -4 & -1 \\ 0 & -3 & 6 & 1 \end{bmatrix}.$$

(3) Give an example of a Hermitian and a Skew-Hermitian matrix of order 3.

5 Attempt any five : **10**

(1) Find $|z|$ and principal Arg. of $z=(1+i)^{1-i}$.

(2) Solve by De-Moiver's theorem :

$x^3-x^2+x-1=0$.

(3) Solve : $\tanh z+2=0$.

(4) Write the P.I. of differen.

$\left(D^3+D^2-D-1\right)y=\cos 2x$.

(5) Obtain the radius of convergence of

$$\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{\frac{n^2}{2}}\cdot x^n.$$

(6) Solve : $p^3-4xyp+8y^2=0$.

(7) Transform the matrix $\begin{bmatrix}2 & -1 & 4\\ 3 & 0 & -1\\ 1 & 0 & 3\end{bmatrix}$ into the Echelon form by row operations.

(8) If A is a square matrix then prove that $A+A^*$ is Hermitian and $A-A^*$ is Skew-Hermitian matrix.
