P.S.SCIENCE & H.D.PATEL ARTS COLLEGE, KADI
INTERNAL EXAMINATION

07/03/2019 | B.Sc. Sem -VI | Marks 40
Number Theory | Time: 1.45 to 3.45
CC-MAT-603 (B)

1. (A) State and Prove Division algorithm

OR

Let a and b are integers, not both zero then $gcd(a,b) = 1$ iff there exist integers x and y such that, $ax + by = 1$

(B) Attempt any two:

1. Show that, cube of any integer has one of the form $9k, 9k+1, 9k+8$.

2. Express lcm(143,227).

3. Divide 100 into two summands such that one is divisible by 7 and other by 11.

2. (A) For arbitrary integers a and b, a $\equiv b\ (mod\ n)$ iff a and b leave the same non negative remainders when divided by n

OR

The linear congruence $ax \equiv b\ (mod\ n)$ has a solution

iff d | b. where d=gcd(a,n)

(B) Attempt any two:

1. Prove that, $89 \mid 2^{44} - 1$

2. Solve: $x \equiv 2\ (mod 3), x \equiv 3\ (mod 5), x \equiv 2\ (mod\ 7)$.

3. For any integer a, show that,

$a^2 - a + 7$ ends in one of the digits 3, 7, 9

3. (A) State and prove Fermat's theorem.

OR

State and prove Wilson's theorem.

(B) Attempt any two:

1. Prove that, if p and p+2 are pair of twin primes then,

$$4((p-1)! + 1) + p \equiv 0 \ (mod\ p\ (p+2))$$

2. Use Fermat's method to factor 10541.
3. Find the last two digits of 9^{9^9}.

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