P.S.SCIENCE & H.D.PATEL ARTS COLLEGE, KADI
INTERNAL EXAMINATION

	B.Sc. Sem -VI	Marks 40
06/03/2019	Mathematics	Time: 1.45 to 3.45
	CC-MATH-602	

1. **(A) Attempt any two.**

(i) Let $f_1, f_2, f_3, \ldots, f_k$ be the real function on a metric space X and Let $\overline{f}$ be the mapping of X into R^k defined by $\overline{f}(x) = (f_1(x), f_2(x), \ldots\ldots, f_k(x)), (x \in X)$ then $\overline{f}$ is continuous if and only if each f_i is continuous where $i = 1, 2, 3, \ldots, k$.

(ii) Prove that a mapping f of a metric space X into metric Y is continuous on X if and only if $f^{-1}(v)$ is open in X, for every open set v in Y.

(iii) Show that continuous image of connected set is connected.

(iv) If f is a real continuous function on $[a, b]$ which is differentiable in (a, b), then there is a point $x \in (a, b)$ at which

$$f(b) - f(a) = (b - a)f'(x)$$

(B) Attempt any one.

(i) Define $f(x) = x + 2 \quad -3 < x < -2$

$$= x + 2 \quad 0 \le x < 1 = -x - 2 \quad -2 \le x < 0$$

Discuss the continuity at $x = 0$.

(ii) Evaluate

$$\lim_{x \to 0} \frac{x - \tan x}{x^3} \text{ and } \lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x}}$$

2. **(A) Attempt any two.**

(i) Prove that every continuous function is R-S integrable

(ii) If $f \in R(\alpha)$ on $[a, b]$ & c is constant then prove that

$(a)\ cf \in R(\alpha)\ on\ [a,b]\ \ (b) \int_a^b cf\, d\alpha = c\int_a^b f d\alpha.$

(iii) State & prove the Fundamental theorem of calculus.

(B) Attempt any one.

(i) Using definition of R-S integral, prove that

$$\int_0^3 x\, d(x-[x]) = -\tfrac{3}{2}$$

(ii)If $f:[0,1] \to R, f(x) = \sqrt{1-x^2},\ if\ x \in Q \cap [0,1]\ and$ $f(x) = 1-x,\ if\ x\ is\ otherwise and\ \alpha(x) = x, x \in [0,1]$ then prove that $f \notin R(\alpha)\ on\ [0,1]$

3. (A) Attempt any two.

(i) If K is a compact metric space if $f_n \in C(K)$, $n = 1,2,3,\ldots$ and if $\{f_n\}$ convergence uniformly on K then $\{f_n\}$ is equi-continuous on K.

(ii) Suppose $f_n \to f$ uniformly on E in a metric space X . Let x be a limit point of E & suppose that $\lim_{n\to\infty} f_n(t) = A_n$ then prove that $\{A_n\}$ convergence & $\lim_{t\to x} f(t) = \lim_{n\to\infty} A_n$.

In other words prove that $\lim_{t\to x} \lim_{n\to\infty} f_n(t) = \lim_{t\to x} \lim_{n\to\infty} f_n(t)$

(iii) Prove that $C(X)$ is complete metric space.

(B) Attempt any one.

(i) *If* $f_n(x) = \dfrac{x^2}{x^2 + (1-nx)^2}$, $0 \le x \le 1,\ n = 1,2,3,\ldots$ *then show that* $\{f_n\}$ *is uniformly bounded on* $[0,1]$ *but not uniformly convergence*

(ii) Give an example of a sequence of functions for which

$$\lim_{n\to\infty} \left[\tfrac{d}{dx} f_n(x)\right] \neq \tfrac{d}{dx}\left[\lim_{n\to\infty} f_n(x)\right]$$
