Pramukh Swami Science & H D Patel Arts College, Kadi
Internal Examination, March-2019,
B.Sc. Semester- VI (Mathematics)
CC MATH-601 Ring theory

Date:05/03/2019
Time: 01:45 to 03:45 **Total Marks: 40**

1. (a) Define characteristic of a ring and Show that, the characteristic of a ring R with unity is n iff n is the smallest positive integer with $n \cdot 1 = 0$. **[06]**

OR

Define Boolean ring and show that, Boolean ring is always commutative ring with characteristic 2.

(b) Attempt any three of the following: **[09]**

1) Let I is an ideal in a ring with unity and $1 \in I$ then show that, $I = R$.
2) Let R be a commutative ring and $a \in R$ then show that, the set $I = \{x \in R \mid ax = 0\}$ is an ideal of ring R.
3) If $x^2 = x, \forall x \in R$, where R is a ring then prove that, R is a commutative ring of characteristic 2.
4) Prove that, a field has no proper ideal.

2. a) State and prove Division algorithm theorem for polynomials. **[07]**

OR

a) Prove that, for an integral domain D, the set D[x] of all polynomials over D is also an integral domain under addition and multiplication of polynomials but D[x] is not a field.

(b) Attempt any two of the following: **[06]**

1) For a non zero polynomials f and g in D[x], Show that, $deg[f \cdot g] = deg[f] + deg[g]$.

2) Let $f = ([3], [0], [1], [0], [0], \dots)$ and $g = ([2], [3], [4], [2], [0], [0], \dots)$ be defined in $Z_5[x]$ then obtain $f + g$ and $f \cdot g$.

3) Define Monic polynomial and show that, the product of two monic polynomials is also Monic ploynomial.

3. (a) An ideal I=$\langle p \rangle$ is a maximal ideal of ring $\langle Z, +, . \rangle$ if and only if p is prime.. **[06]**

OR

An Ideal I in a commutative ring with unity is a prime ideal iff the quotient ring R/I is an integral domain.

(b) Attempt any two of the following: **[06]**

1) Show that ideal I=$\langle x^3\text{-}x\text{-}1 \rangle$ is a maximal ideal in $Z_3[x]$.
2) Show that, Homomorphic image of ideal is also ideal.
3) A homomorphism defined on the ring (Z, +, .) is either zero homomorphism or identity mapping.
4) Give an example of an ideal which is Maximal ideal but not a prime ideal.

BEST OF LUCK