

P.S.SCIENCE & H.D.PATEL ARTS COLLEGE, KADI  
INTERNAL EXAMINATION

B.Sc. Sem -IV

Marks 40

06/03/2019

Mathematics

Time: 1.45 to 3.45

CC-MATH- 401

1. (A) Attempt any one.

(i) Obtain the formula for radius of curvature of the curve

$$p = f(r).$$

(ii) Prove that  $\sqrt[n]{n} \sqrt[n+1]{n+1} = \frac{\sqrt{\pi} \sqrt{2n}}{2^{2n-1}}$ .

(B) Attempt any two.

(i) Find the radius of curvature of the curve

$$x^{2/3} + y^{2/3} = a^{2/3}$$

(ii) Evaluate  $\int_0^1 x^5(1-x^3)dx$  and  $\int_0^{\infty} \frac{1}{1+y^4} dy$

(iii) Find the radius of curvature of the curve

$$x^3 + y^3 = 3axy$$

2. (A) Attempt any one.

(i) Change the order of integration  $\int_0^3 \int_{4y/3}^{\sqrt{25-y^2}} f(x,y) dy dx$ .

(ii) Evaluate  $\iint_S \sqrt{xy(1-x-y)} dx dy$  by transforming

into new variables  $u, v$  by equations  $x+y=u, x=uv$ ,

where  $S$  is the region bounded by lines

$x=0, y=0, x+y=1$  in  $XY$ -plane

(B) Attempt any two.

(i) Find the volume of  $x^2 + y^2 + z^2 = a^2$ .

(ii) Find  $\iint (x^2 + y^2) dx dy$  over the region bounded by

$$x=1, x=2, y=1, y=x^2$$

(iii)  $\iiint_V \frac{1}{(x+y+z+1)^3} dx dy dz$ , where  $V$  is enclosed by the

planes  $x=0, y=0, z=0$  and  $x+y+z=1$

3. (A) Attempt any one

(i) State and prove Green's Theorem.

(ii) Prove that

$$\text{curl}(\phi f) = \phi \text{curl} f + (\text{grad } \phi) \times f$$

(B) Attempt any two.

(i) Verify Green's theorem

$\oint_c y^2 dx + x dy$ , where  $c$  is the boundary of the square whose vertices are  $(0,0), (1,0), (1,1), (0,1)$ .

(ii) If  $f = (2yz, -x^2y, xz^2)$  and  $\phi(x, y, z) = 2x^2yz^3$  then

$$(f \cdot \nabla)\phi = f \cdot (\nabla\phi).$$

(iii) Show that,  $\nabla \cdot \left[ r \nabla \left( \frac{1}{r^3} \right) \right] = \frac{3}{r^4}$ , where  $x^2 + y^2 + z^2 = r^2$ .