BC-218

BC-218 Seat No.__________

B. Sc. (Sem. VI) Examination

March/April – 2014

Mathematics : CC - MATH - 603 B

(Number Theory)

Time : 3 Hours] [Total Marks : 70

Instructions: (1) All four questions are compulsory.

(2) Figures to the right indicate marks of the question.

1. Attempt any three of the following. **18**

(a) State and prove Division Algorithm

(b) Prove Binomial theorem by induction

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n}b^n$$

(c) For $n \geq 1$, prove that $n(n+1)(2n+1)/6$ is an integer.

(d) Solve the Diophantine equation $54x + 21y = 906$.

(e) Find the integers x and y satisfying $\gcd(1769,2378) = 1769x + 2378y$ by Euclidean Algorithm.

2. Attempt any three of the following. **18**

(a) For arbitrary integers a and b, $a \equiv b(mod\ n)$ if and only if a and b leave the same nonnegative remainder when divided by n.

(b) An integer $b > 1$ any positive integer N can be written as

$$N = a_m b^m + a_{m-1} b^{m-1} + \cdots + a_2 b^2 + a_1 b + a_0 \text{ uniquely.}$$

(c) Determine last three digits of the number 7^{999}.

(d) Let $n > 0$ be fixed and a, b, c be arbitrary integers and if $a \equiv b(mod\ n)$ then (i) $a + c \equiv b + c(mod\ n)$ (ii) $ac \equiv bc(mod\ n)$.

(e) Prove any prime of the form $3n + 1$ is also of the form $6m + 1$.

3. Attempt any three of the following **18**

(a) State and prove Wilson's Theorem.

(b) If the integer $n > 1$ has the prime factorization $n = {p_1}^{k_1}.{p_2}^{k_2} \cdots {p_r}^{k_r}$, then

$\phi(n) = n(1 - {}^1/_{p_1})(1 - {}^1/_{p_2}) \cdots (1 - {}^1/_{p_r})$.

(c) Find the unit digit of 3^{100} by means of Euler's theorem.

(d) Find the factor of the number 10541 by Fermat's theorem.

(e) State Chinese Remainder Theorem and Solve: $2x \equiv 1$ (mod 3), $3x \equiv 1$(mod 5) and $5x \equiv 1$(mod 7).

4. Attempt any four of the following **16**

(a) Define l. c. m. and g. c. d. of integers a and b.

(b) If a and b are given integers, not both zero, then a and b are relatively prime if and only if there exist integers x and y such that $1 = ax + by$.

(c) Find the remainder when $1! + 2! + 3! + \cdots + 100!$ is divisible by 12.

(d) Verify $\phi(n) = \phi(n + 1) = \phi(n + 2)$ holds when $n = 5186$.

(e) Solve the linear congruence $17x \equiv 9(mod\ 276)$.
