



BS-1363

Seat No. _____

B. Sc. (Sem. IV) Examination

March / April - 2014

CC MATH-402 : Advanced Linear Algebra

Time : 3 Hours]

[Total Marks : 70

- Instructions :** (i) All questions are compulsory.
(ii) Figure in the right side indicate the marks of the question.

- 1 (a) Prove that an $n \times n$ matrix A is invertible 8
if and only if the corresponding linear map
 T of A with respect to the standard basis is
non-singular.
- (b) Attempt any two : 12
- (i) Obtain the linear transformation associated

with the matrix $A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 0 & 1 & -1 \\ 1 & 2 & 0 & 0 \end{bmatrix}$ by

taking the standard basis of R^3 and R^4 .

(ii) $T: R^3 \rightarrow R^2$ and

$$T(e_1) = (2, -2), T(e_2) = (1, 2), T(e_3) = (0, 0)$$

$$B_1 = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\} \text{ and}$$

$$B_2 = \{(1, 1), (1, -1)\} \text{ are basis of } R^3 \text{ and}$$

R^2 respectively.

Find $[T: B_1, B_2]$.

(iii) Find range, null space and rank, nullify

$$\text{of matrix } A = \begin{bmatrix} 5 & 0 & 1 & 6 \\ 0 & 0 & 3 & 3 \\ 1 & 0 & -1 & 0 \end{bmatrix}.$$

2 (a) State and prove triangular inequality. 8

OR

(a) Let V be a vector space which is an inner product space then $|\langle x, y \rangle| = \|x\| \|y\|$ iff x and y are linearly dependent. 8

(b) Attempt any two : 12

(i) Using Gram-Schmidt process obtain the orthonormal basis from the basis

$$\{(-1, 1, 1), (1, -1, 1), (1, 1, -1)\}.$$

(ii) Define an inner product space. In a vector space R^2 for $x = (x_1, x_2)$,

$$y = (y_1, y_2) \text{ define}$$

$$\langle X, Y \rangle = x_1 y_1 + x_1 y_2 + x_2 y_1 - 3x_2 y_2 \text{ then show that } \langle X, Y \rangle \text{ is an inner product in } R^2.$$

(iii) Define a dual basis. Find dual basis for the base $\{(1, -1, 3), (0, -1, -1), (0, 3, -2)\}$.

3 (a) State and prove Cayley-Hamilton theorem. 8

OR

(a) Prove that eigen vectors of symmetric linear map corresponding to different eigen values are perpendicular to each other. 8

(b) Attempt any two : 12

(i) Using Cayley-Hamilton theorem find the

$$\text{inverse of matrix } A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & 1 \\ 0 & 5 & 3 \end{bmatrix}.$$

(ii) Find the eigen values and corresponding

eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

(iii) Find the minimal polynomial for matrix

$$A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}.$$

4 Attempt any four :

10

(i) Prove that $(AB)^T = B^T A^T$.

(ii) For a linear map

$$T: R^2 \rightarrow R^2 \quad T(\alpha, \beta) = (2\alpha + \beta, \alpha + 3\beta) \quad \text{find } T^*.$$

(iii) State and prove parallelogram law.

(iv) Solve the system of following equation by using Echelon matrix.

$$2x + 3y - z = 1, \quad x - 3y + 2z = -1, \quad x - y + 2z = 2$$

(v) Define :

(i) inner product space

(ii) minimal polynomial