BS-1359 Seat No.________

B. Sc. (Sem. - IV) Examination

March / April - 2014

CCMAT-401 - Mathematics

(Advanced Calculus)

Time : 3 Hours] [Total Marks : 70

1 (a) Find out radius of curvature of curve $r = f(\theta)$. 7

OR

(a) Find out radius of curvature of curve $p = f(\psi)$. 7

(b) Attempt any **two** : 8

(1) Find out radius of curvature of $r = a(1+\cos\theta)$.

(2) Find the centre of curvature of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; where $a > b > 0$.

(3) Find the double points of the curve : $xy^2 - ax^2 + 2a^2x - a^3 = 0$

2 (a) Prove that $\beta(m,n) = 2\int_0^{\pi/2} \sin_\theta^{2m-1} \cdot \cos_\theta^{2n-1} \, d\theta$ 7

OR

(a) Prove that : 7

$$\Gamma n = \frac{1}{n}\int_0^\infty e^{-x^{\frac{1}{n}}}\, dx; \text{ where } n > 0.$$

(b) Attempt any **two** : 8

(1) Evaluate : $\int_0^1 x^5\left(1-x^3\right)^{10} dx$

(2) Evaluate : $\int_0^1 \sqrt{1-x^4}\, dx$

(3) Prove that : $\int_0^1 \frac{1}{\sqrt{\log_e\left(\frac{1}{x}\right)}} dx = \sqrt{\pi}$

3 (a) Evaluate : $\iint_S \left(x^2+y^2\right) dxdy$ 7

Where S is the region bounded by $x=1, x=2,\ y=1,\ y=x^2$.

OR

(a) Using polar co-ordinate, evaluate : 7

$$\iint_S \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}\, dxdy$$

Where $s=\left\{(x,y)/x\ge 0; y\ge 0; x^2+y^2\le 1\right\}$

(b) Attempt any two : 8

(1) Evaluate :

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^b \left(y^2+z^2\right) dx\, dy\, dz$$

(2) Evaluate

$$\int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} \sqrt{\frac{a^2b^2-b^2x^2-a^2y^2}{a^2b^2+b^2x^2+a^2y^2}}\, dx\, dy$$

(3) Change the order of integration

$$\int_0^{2a} \int_{\frac{x^2}{4a}}^{3a-x} f(x, y)\, dx\, dy$$

4 (a) State and prove : Stoke's theorem. 7

OR

(a) State and prove : Green's theorem.

(b) Attempt any two : 8

(1) Evaluate : $\oint (\sin z\, dx - \cos x\, dy + \sin y\, dz)$

where $C = \{(x, y, z)/0 \le x \le \pi; 0 \le y \le 1; z = 3\}$

(2) Verify the divergence theorem

$$\iint_S \left[\left(x^3 - yz\right) dy\, dz - 2x^2 y\, dz\, dx + z\, dx\, dy\right]$$

Where S is a surface of the cube with faces :

$x = 0,\ x = a,\ y = 0,\ y = a,\ z = 0,\ z = a$

(3) Prove that : Curl $\left(grad\ r^n\right) = \overline{0}$;

where $r^2 = x^2 + y^2 + z^2$; $\overline{r} = (x, y, z) \in R^3$

5 Attempt any five : 10

(1) Find the radii of curvature of the curve

$x^2 + 2xy + 2y^2 - 4x = 0$ at origin

(2) Evaluate : $\int_0^4 \int_0^{\sqrt{16-x^2}} xy\, dx\, dy$

(3) Change the order of $\int_{0}^{2a} \int_{x^2/4a}^{3a-x} f\,dx\,dy$

(4) Evaluate : $\int_{0}^{\infty} e^{-x^2}\,dx$

(5) If $\bar{r} = (x, y, z) \in R^3$; $|\bar{r}| = r$; then show that :

$div\left(r^m \bar{r}\right) = (m+3)\,r^m$; where $m \in N$.

(6) If $\bar{F} = \left(xy^2z,\ -x^3y^3,\ x^2yz^3\right)$; then Curl $\bar{F} = \bar{0}$.
