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MAV-3457 Seat No._____

M. Sc. (Sem. III) Examination

October / November - 2018

Mathematics : MTHP - 5

(Advanced Topology)

Time : 3 Hours] [Total Marks : 90

1 Attempt any **three** of the following : 18

(a) Let A be a subset of a topological space X. If there is a sequence of a points of A converging to x, prove that $x \in \overline{A}$. When converse is hold ? Explain.

(b) Is the space $\mathbb{R}_l$ a Lindelöff space ? Explain.

(c) Prove that a subspace of first countable space is first countable and a subspace of second countable space is second countable.

(d) Prove that every second countable space is separable.

(e) Prove or disprove : The product of two Lindelöff spaces is Lindelöff.

2 Attempt any **three** of the following : 18

(a) Prove that every regular space is a Hausdorff space. Does the converse hold ? Justify your answer.

(b) Show that if X is normal, every pair of disjoint closed sets have neighborhoods whose closures are disjoint.

(c) Prove that every metrizable space is normal.

(d) Show that a closed subspace of a normal space is normal.

(e) Show that every locally compact Hausdorff space is completely regular.

3 Attempt any **two** of the following : 18

(a) Prove that for any open covering $\{U_1, U_2 ..... U_n\}$ of a normal space X there exists a partition of unity dominated by $\{U_i\}$.

(b) If X is compact m-manifold, prove that X can be imbedded in $\mathbb{R}^N$ for some positive integer N.

(c) What is compactification ? Give illustration. If topological space X has a compactification, show that X must be completely regular.

4 Attempt any **three** of the following : 18

(a) In a metric space X if every Cauchy sequence has a convergent subsequence, prove that X is complete.

(b) Prove that every complete and totally bounded metric space is compact.

(c) Let $C_1 \supset C_2 \supset ....$ be a nested sequence of nonempty closed sets in the complete metric space X. If $\text{diam}\, C_n \to 0$, prove that $\cap C_n \neq \phi$.

(d) Let X equal the countable union $\cup B_n$. Show that if X is a nonempty Baire space, at least one of the sets $\overline{B_n}$ has a nonempty interior.

(e) Prove that any open subspace Y of a Baire space X is itself a Baire space.

5 Attempt any **six** of the following : **18**

(a) Define first countability and second countability axiom in a topological space.

(b) Prove or disprove : Every first countable space is second countable.

(c) Let X and X' denote a single set under two topologies T and T' respectively; assume that $T' \supset T$. If one of the spaces is normal, what does that imply about the other?

(d) State Urysohn lemma. (Do not prove)

(e) State Tietze extension theorem. (Do not prove)

(f) Let $A \subset X$; let $f : A \to Z$ be continuous map of A into the Hausdorff space Z. Prove that there is at most one extension of f to a continuous function $g : \overline{A} \to Z$.

(g) Give an example of a metric space which is not complete. Explain.

(h) State Ascoli's theorem. (Do not prove.)
