



MAV-3504

Seat No. _____

M. Sc. (Sem. III) Examination

October / November - 2018

Mathematics : MTHE-C-1

(Mathematical Modeling)

Time : 3 Hours]

[Total Marks : 70

Instructions : (1) All questions are compulsory and carry equal marks.

(2) Standard notations and usual conventions are followed.

1 Attempt any **two** : 14

(a) State the Logistic law. Establish the logistic population growth model. Determine the limiting value. Interpret your result.

(b) Explain the population growth model. Describe the effect of immigration and emigration on the population.

(c) Using dimensional analysis: Find a model determining the terminal velocity of a rain drop falling from motionless cloud.

2 (a) Write short notes on the followings : 4

(1) Phases of model building.

(2) Classification of Models.

(b) Discuss the stability of the equilibrium 10

position $(0,0)$ and $(C/D, A/B)$ for the prey-predator model represented by the equations

$$\frac{dy}{dx} = Ay - Bxy, \quad \frac{dx}{dt} = -Cx + Dxy$$

OR

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- (b) Define the following terms: Infective, Susceptible, Carrier. Study the spread of an epidemic by constructing appropriate differential equations. State your assumptions clearly. If we assume that an infective recovers and becomes susceptible, how would the result change ? 10

3 Attempt any two : 14

- (a) State continuity principle. Using this derive a model to detect the concentration of medicine in the blood stream. Find the formula for maximum as well as minimum level of concentration.
- (b) "If any population is crossed at random with only hybrids successively then recessive never appears": Justify the statement, modeling point of view.
- (c) "It is advisable that: After 4 hours food is essential for diabetes patient,": Derive the diabetes mellitus model with this constraints.

4 (a) Derive a microscopic car-following non-linear model. 14

- (b) Derive a fundamental diagram of a road traffic using macroscopic traffic flow model. How would you determine the capacity of a given highway?

OR

- 4 (a) Derive a model for maximizing flow on narrow bridges. 14
- (b) Using "Principle of conservation of cars" establish the relationship among three fundamental traffic variables.
- 5 Write any five answer from the followings : 14
- (a) Why modeling is a natural human activity? Explain.
- (b) Suppose we have 50% plants with red flowers, 20% plants with pink flowers and 30%, plants with white flowers. Under random crossing hypotheses, what will be the percentage in the next succeeding generations ?
- (c) Explain the following terms in sense of "Traffic Models":
Density of Traffic (ρ), velocity of traffic (v), flow of traffic (q). Derive that $q = v\rho$
- (d) A bacterial population has a doubling time of 20 minutes. Starting with a population of 10 bacteria, what would be the potential population size after 12 hours ?
- (e) Mr. X and his daughter both have blood group B, his father both have group A. What are the possible blood groups of later children and with what probability ?
- (f) Explain the followings with details :
- (1) What is Mathematical Modeling ?
- (2) What are the scope and limitation of it ?

(g) 100 km long super express highway is to be constructed from Patan to Ambaji with expected speed limit 140 km/hr. In 2025, expected number of vehicles will enter in this highway in each direction will enter in this highway in each direction is 6000; in 2030 expected vehicles are 12000 and 2035 it becomes 20,000. How many lanes should be required ? Assuming that the safe distance required 10 mt. for 100 km/hr speed.

(h) Consider all the families with two children in which one parent has genotype AA and the other parent Aa. Prove that,

- $\frac{1}{4}$ of the families have two AA children.

- $\frac{1}{2}$ of the families have one AA and one Aa children

- $\frac{1}{4}$ of the families have two Aa children.