MEB-4738 Seat No.________

M. Sc. (Sem. I) (Mathematics) Examination

November / December - 2018

MCB - 1 : Special Functions

Time : 2 Hours] [Total Marks : 35

Instructions : (1) All questions are compulsory.
(2) Follow the standard notations and conventions.

1 (a) Consider the equation **5**

$$x(1-x)y'' + \{c-(a+b+1)x\}y' - a\,b\,y = 0.$$

(i) Determine the singular points. Are they regular ? Explain.

(ii) Obtain the general solution of this equation near the point $x = 0$.

(iii) Find the second solution about the origin in terms of $F(a,b,c;x)$.

OR

(a) Do the followings : **5**

(i) Obtain the general solution of Gauss's Hyper geometric equation near $x = 1$.

(ii) Find the general solution of $\left(1-e^{x}\right)y'' + \frac{1}{2}y' + e^{x}y = 0$ near the singular point $x = 0$ by changing the independent variable to $t = e^{x}$.

(b) Attempt any **one** : **2**

(i) Prove that $\log(1+x) = x\,F(1, 1, 2; -x)$.

(ii) Prove that

$$F'(a, b, c; x) = ab/cF(a+1, b+1, c+1; x)$$

2 (a) For Chebyshev's polynomial prove that **5**

$$T_n(x) - \sum_{r=0}^{[n/2]} \frac{(-1)^n n}{(2r)!(n-2r)!}\left(1-x^2\right)^r (x)^{n-2r}$$

OR

(a) Derive Hermite polynomial $H_n(x)$ from solving Hermite equation $y'' - 2xy' + 2p\,y = 0$.

Show that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$ is a generating function for $H_n(x)$.

(b) Attempt any **one** : **2**

(1) Show that

$$T_n(x) = \frac{1}{2}\left[\left(x - \sqrt{x^2-1}\right)^n + \left(x + \sqrt{x^2-1}\right)^n\right]$$

(2) Show that $T_{n+1}(x) - 2T_{n+1}(x) + T_{n-1}(x) = 0$

3 (a) Obtain Legender polynomial from solving Legender's differential equation $\left(1-x^2\right)y'' - \lambda xy' + p(p+1)y = 0$, where p is constant. **5**

OR

(a) Show that $\left\{1 - 2xt + x^2\right\}^{-1/2} = \sum_{n-0}^{\infty} t^n p_n(x)$ **5**

is the generating function for Legender polynomial.

(b) Attempt any **one** : **2**

(1) Show that

$$(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$$

(2) Show that

$P_n(1) = 1, P_n(-1) = (-1)^n, P_{2m}(0) = 0$

and find $P_{2m+1}(0)$.

4 (a) Do the following : 5

(i) Find the general recurrence relation term using the indicial equation for Bessel's equation of order p, $x^2 y'' + x y' + (x^2 - p^2) y = 0$. What problem occurs whenever one of the roots of the initial equation is a negative, even integer?

(ii) Define Bessel function $J_p(x)$ and state how they arise from Bessel's equation.

OR

(a) Obtain the general solution of Bessel's differential equation 5

$x^2 y'' + x y' + (x^2 - p^2) y - 0$, where p is constant..

(b) Attempt any **one** : 2

(1) Prove that $\frac{d}{dx} J_o(x) = -J_1(x)$ and $\frac{d}{dx}(x J_1(x)) = x J_0(x)$ for $n \in N$.

(2) Prove that $I_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ and

$I_{1/-2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$

4 (a) Attempt any **two** : 4

(1) Show that the point $x=1$ is a singular point of $y''+(\cos \pi x)y'+y/(1-x)=0$ but the point $x=0$ is an ordinary point of the same equation.

(2) Show that the indicial equation for $y''+P(x)y'+Q(x)y=0$ is $\rho(\rho-1)+p_o\rho+q_0=0$, assuming that origin is the regular singular point of the equation. Also explain the Frobenius method to find the solution of a second order ODE

(3) Explain the Frobenius method to find the solution of a second order ODE $y''+P(x)y'+Q(x)y=0$. Also explain the method of find second solution when two roots of the initial equation are equal or differ by and integer.

(b) Find Chebyshev polynomial / Hermit's polynomials $T_0(x), T_1(x), T_2(x)$ and $T_3(x)$. 3