



**MEB-4705**

Seat No. \_\_\_\_\_

**M. Sc. (Sem. I) Examination**

**November / December - 2018**

**Mathematics : Paper - MTHP - 4**

*(General Topology)*

Time : 3 Hours]

[Total Marks : 90

**Instructions :**

- (1) All questions are **compulsory**.
- (2) Each question carries **equal** marks.
- (3) Follow the standard notations and conventions.

1 Attempt any **three** :

18

- (a) Prove : The standard topology on  $\mathbb{R}$  is an order topology derived from the usual order on  $\mathbb{R}$ .
- (b) Prove : The collection

$$\mathcal{J} = \left\{ \pi_1^{-1}(U) / U \text{ open in } X \right\}$$

$$U \left\{ \pi_1^{-1}(V) / V \text{ open in } Y \right\}$$

is a sub basis for the product topology on  $X \times Y$ .

- (c) Prove : Let  $Y$  be a subspace of  $X$ . Then a set  $A$  is closed in  $Y$  iff there exists closed set  $C$  in  $X$  s.t.  $A = C \cap Y$ .
- (d) If  $A$  be a subset of topological space  $X$  and if  $A'$  be the set of all limit points of  $A$ ; then show that  $\bar{A} = A \cup A'$ .

- (e) Prove : Let  $X$  be a space satisfying the  $T_1$ -axiom. Let  $A$  be a subset of  $X$ .

If  $x \in X$  is a limit point of  $A$ ; then every neighborhood of  $x$  contains infinitely many points of  $A$ .

2 Do any **three** : 18

- (a) State and Prove : The pasting lemma.  
(b) State and Prove : The sequence lemma.  
(c) If  $X, Y$  and  $Z$  be topological spaces and if  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  are continuous; then show that  $g \circ f : X \rightarrow Z$  is continuous.  
(d) Prove : The topology  $T_d$  on  $\mathbb{R}^n$  induced by the euclidean metric  $d$  and the topology  $T_p$  induced by the square metric  $p$  are the same.  
(e) Show that : The function  $F : (-1, 1) \rightarrow \mathbb{R}$

defined by  $F(x) = \frac{x}{1-x^2}$  is a

homeomorphism.

3 Attempt any **three** : 18

- (a) Prove : Connectedness is a topological property.  
(b) Prove : If  $X$  and  $Y$  are connected topological spaces; then  $X \times Y$  is also connected topological space.  
(c) Prove : A path - connected space is connected. Give only example of a connected space - which is not a path - connected.

- (d) Prove : The components of  $X$  are connected disjoint subspaces of  $X$  whose union is  $X$  such that each non empty connected subspace of  $X$  intersects only one of them.
- (e) Prove : If for every open set  $U$  of  $X$ ; each component of  $U$  is open in  $X$ ; then space  $X$  is locally connected.

4 Do any **three** : 18

- (a) Prove : Every closed subspace of a compact space is compact.
- (b) State and Prove : The Tube Lemma.
- (c) Prove : Subspace  $A$  of  $\mathbb{R}^n$  is compact iff it is closed and bounded in the euclidean metric  $d$  or the square metric  $P$ .
- (d) State and Prove : Extreme Value Theorem.
- (e) Show that : Closed subset of a locally compact Hausdorff space is locally compact.

5 Attempt any **six** : 18

- (i) Let  $B = \{ \{a, b\}, \{b, c\}, \{c, d, e\} \}$  be a collection of subsets of  $X = \{a, b, c, d, e\}$ . Is  $B$  basis for a topology on  $X$  ? Why ?
- (ii) Show that : The order topology on  $Z_+ = \{1, 2, 3, \dots\}$  is the discrete topology on  $Z_+$ .

(iii) Let  $T_Y = \{ \phi, Y, \{b\} \}$  be a topology on  $Y = \{a, b\}$ ; and  $T_X = \{ \phi, X, \{1\}, \{2, 3\} \}$  be a topology on  $X = \{1, 2, 3\}$ . Let  $f : X \rightarrow Y$  defined by  $f(1) = a; f(2) = f(3) = b$ . Is  $f$  continuous on  $X$ ? Why?

(iv) Let  $(X, d)$  is a metric space; Where  $d : X \times X \rightarrow \mathbb{R}$  defined as

$$d(x, y) = \begin{cases} 1; & \text{if } x \neq y \\ 0; & \text{if } x = y. \end{cases}$$

Then show that : Metric topology induced by  $d$  is the discrete topology on  $X$ .

(v) Give an example of a disconnected space whose at least one subset is connected.

(vi) Show that : Punctured euclidean space  $\mathbb{R}^n - \{0\}$  (Where  $n > 1$ ) is a path - connected space.

(vii) Show that an infinite subset  $A$  of a discrete topological space  $(X, 2^X)$  is not compact.

(viii) If  $T$  is a cofinite topology on  $X$ ; then show that  $(X, T)$  is compact.