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MEB-4714

Seat No.________

M. Sc. (Sem. I) Examination

November / December - 2018

MTHE - A-2 : Mathematics

(Techniques of Differential Equation)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Follow the usual notations and conventions.

1 (a) Prove necessary and sufficient condition that the differential equation $Pdx + Qdy + Rdz = 0$ is integrable. **8**

OR

(a) Derive the Natani's method to solve the differential equation $Pdx + Qdy + Rdz = 0$ **8**

(b) Solve any one of the following : **7**

(1) $\left(y^2 + yz + z^2\right)dx + \left(z^2 + zx + x^2\right)dy + \left(x^2 + xy + y^2\right)dz = 0$

(2) $2y(a-x)dx + \left[z - y^2 + (a-x)^2\right]dy - ydz = 0$

2 (a) Derive the formula for Charpit's Method to solve the differential equation $f(x, y, z, p, q) = 0$ **8**

OR

(a) Obtain the formula of orthogonal surfaces to a system of surfaces $f(x,y,z)=0$ **8**

(b) Attempt any **one** of the following : **7**

(1) Solve $(y+zx)p-(x+yz)q=x^2-y^2$

(2) Find integral surface of $(x-y)y^2p+(y-x)x^2q=\left(x^2+y^2\right)z$ passing through $xz=a^2, y=0$

3 Attempt any **two** : **14**

(1) If $(\alpha D+\beta D'+\gamma)$ is a factor of $F(D,D')z=0$ and $\phi(\xi)$ is an arbitrary function of single variable ξ then prove that $u=\exp\left(\dfrac{-\gamma x}{\alpha}\right).\phi(\beta x-\alpha y)$ is solution of $F(D,D')z=0, (\alpha\neq 0)$

(2) Solve $\left(D^2+DD'+D'-1\right)z=\sin(x+2y)$.

(3) Find the surface passing through the two lies $z=x=0, z-1=x-y=0$ satisfying the equation $r-4s+4t=0$.

4 Attempt any **two** : **14**

(1) State and solve the Neumann problem in rectangle.

(2) Solve $\dfrac{\partial u}{\partial x}=2\dfrac{\partial u}{\partial t}+u$ where $u(x,0)=6e^{-3x}$

(3) Derive the D' Alembert solution of one dimensional Wave equation.

5 Attempt any **four** : **12**

(1) Is the differential equation $\left(y^2 + xz\right)dx + \left(x^2 + yz\right)dy + 3z^2dz = 0$ integrable ? Justify your answer.

(2) Find the particular integral of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$

(3) Solve the differential equation $p^2 + q^2 = 1$

(4) Solve $\left(D^{2-}D'^2 + D - D'\right)z = 0$

(5) Show that the differential equation $z = px + qy$ and $2xy\left(p^2 + q^2\right) = z\left(yp + xq\right)$ are compatible