MAF-761

MAF-761 Seat No.________

B. Sc. (Sem. V) Examination
October / November – 2018
Mathematics : Paper-CCMATH : 502
(Mathematical Analysis-1)

Time : 3 Hours] [Total Marks : 70

Instructions :

(1) All questions are compulsory.

(2) Follow the standard notations and conventions.

1 (a) State and Prove : Archimedean property of $\mathbb{R}$.

(b) State and Prove : Schwarz ineqality for complex numbers.

(c) Prove : The ordered set $\mathbb{R}$ has the least-upper-bound property.

OR

1 (a) Prove : $\forall x \in \mathbb{R}^+$ and $\forall n \in N$; there exists one and only one real y s.t. $y^n = x$.

(b) Prove : Q is dense in $\mathbb{R}$.

(c) Prove : For a fix $\alpha \in \mathbb{R}$ and for $\beta = \{P \in Q / -p - r \notin \alpha;$ for some $r > 0\}$; $\alpha + \beta = 0^*$.

2 (a) Prove : Countable union of a countable set is also countable.

(b) Prove : Compact subsets of metric spaces are closed.

(c) Prove : $E C \mathbb{R}^1$ is connected iff if $x \in E, y \in E$ and $x < z < y$; then $z \in E$.

OR

2 (a) Prove : If P be a nonempty perfect set in $\mathbb{R}^k$; then P is uncountable.

(b) Prove : Every K-cell is compact.

(c) Prove : If P is a limit point of E; then every neighborhood of P contains infinitely many points of E.

3 (a) Let $\{P_n\}_{n=1}^{\infty}$ be a sequence in a metric space X.

Prove : $\{P_n\}_{n=1}^{\infty}$ converges to $P \in X$ iff every neighborhood of P contains all but finitely many of the terms of $\{P_n\}_{n=1}^{\infty}$.

(b) Prove : If X is a compact metric space and if $\{P_n\}_{n=1}^{\infty}$ is a Cauchy sequence in X; then $\{P_n\}_{n=1}^{\infty}$ converges to some point of X.

(c) If $P \in \mathbb{R}^+$ and $\alpha \in \mathbb{R}$, then prove

$$\lim_{x \to \infty} \frac{n^{\alpha}}{(1+P)^n} = 0.$$

OR

3 (a) State and prove : ROOT TEST.

(b) State and prove : Cauchy's condensation Test.

(c) Show that : $e \notin Q$; where Q = The set of all rational numbers.

4 Attempt any **two** :

(a) Prove : The set of all subsequential limits of a sequence $\{P_n\}_{n=1}^{\infty}$ in a metric space X form a closed subset of X.

(b) Prove : In $\mathbb{R}^k$ every Cauchy sequence converges.

(c) Prove : $\sum \frac{1}{nP} = \begin{cases} \text{converges; if } P > 1 \\ \text{diverges ; if } P \le 1 \end{cases}$.

5 Attempt any **two** :

(a) If $\bar{a} \in \mathbb{R}^k, \bar{b} \in \mathbb{R}^k$; then find $\bar{c} \in \mathbb{R}^k$ and $r > 0$ such that $|\bar{x} - \bar{a}| = 2|\bar{x} - \bar{b}|$ iff $|\bar{x} - \bar{c}| = r$.

(b) Let d : IR × IR → defined by $d(x, y) = \frac{|x-y|}{1+|x-y|}$. Is d metric on $\mathbb{R}$? Why ?

(c) Discuss the convergence of the following series :

(i) $\frac{1}{5} + \frac{1}{6} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{5^3} + \frac{1}{6^3} + \ldots\ldots\ldots\ldots$

(ii) $\sum \frac{n^3}{3^n} z^n$
