MAE-676 Seat No.________

B. Sc. (Sem. III) Examination

October / November - 2018

Mathematics : Paper - CCMATH-302

(Numerical Analysis)

Time : 3 Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.
(2) The figures to the right indicate the marks of the corresponding question.
(3) Follow the standard notations and conventions.

1 (a) Define descending differences. 6
State and prove : Gregory- Newton's forward interpolation formula.

(b) Find out $f(6)$; where $f(0)=-3, f(1)=6;$ 6
$f(2)=8;\ f(3)=12$ and third forward differences are constant.

(c) Express $P(x)=x^3+4x^2-7x-15$ in the 6
terms of $(x-2)^{[r]}$; where $h=2$

OR

1 (a) Define ascending differences.
State and prove : Gregory-Newton's backward interpolation formula.

(b) Prove :

$$Y_m + \binom{n}{1} \Delta Y_{m-1} + \binom{n+1}{2} \Delta^2 Y_{m-2} + \ldots\ldots = Y_{n+m};$$

where m, $n \in N$.

(c) Prove :

$$\Delta u^{[k]} = k\, u^{[k-1]} \text{ and } \Delta u^{[-k]} = -k\, u^{[-k-1]};$$

where $k \in N$.

2 (a) State and prove : Gauss backward interpolation formula. 6

(b) State and prove : Lagrange's interpolation formula for unequal intervals. 6

(c) Prove : 6

$$\underset{b,\,c,\,d}{\Delta^2} \left(\frac{1}{2}\right) = -\frac{1}{abcd}$$

OR

2 (a) State and prove : Stirling's interpolation formula.

(b) If $f(x)$ is a polynomial of n[th] degree; then prove that the n[th] divided difference of $f(x)$ is constant.

(c) Prove :

$$f(x_0, x_0, x_0) = \frac{1}{2!} f''(x_0)$$

3 (a) Discuss Picard's method for solving the differential equation. **6**

$$\frac{dy}{dx} = f(x, y); \text{ where } y(x_0) = y .$$

(b) State and prove : Trapezoidal Rule. **6**

(c) **6**

x	0	1	2	3	4
$f(x)$	1	2.72	7.39	20.09	54.60

Then evaluate $\int_0^4 f(x)\,dx$ by Simpson's $\frac{1}{3}^{rd}$ Rule.

OR

3 (a) Discuss Taylor's method for solving the differential equation $\frac{dy}{dx} = f(x, y)$; where $y(x_0) = y_0$.

(b) State and prove : Simpson's $\frac{3}{8}^{th}$ Rule.

(c) Prove : $Q_{31}{}^{(1)} = \frac{h}{24}\{-1, 13, 13, -1\}$.

4 Attempt any two : **8**

(a) State and prove : Simpson's $\frac{1}{3}^{rd}$ Rule.

(b) Prove :

$$\frac{u^{[n]}}{u^{[m]}} = (u-m)^{[n-m]}; \text{ where } h=1; m, n \in Z$$

(c) Prove : $\int_{x_0}^{x_4} y d_x = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right);$

Where $y_r = f\left(x_r\right),\ x_r = x_0 + rh$.

5 Attempt any two : **8**

(a) $Q_{32}(0) = \int_0^2 f(x)dx = \frac{1}{6}\left[8f(0) + 17f(1) - 5f(2) + 3f(3)\right]$

(b) Prove : $\mu^2 = 1 + \frac{1}{4}\delta^2$.

(c) Prove : $\Delta(fg)(x) = f(x)\Delta g(x) + g(x+h)\Delta f(x)$
