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MDD-4264 Seat No. ______

B. Sc. (Sem. I) Examination
November / December - 2018
Mathematics : CC - MATH - 111

Time : **3** Hours] [Total Marks : **70**

Instructions: (1) All questions are compulsory.

(2) Figure to the right indicates the marks of the corresponding question.

1 **(a)** State and prove Liebnitz's Theorem. **06**

OR

(a) State and prove Cauchy's theorem. **06**

(b) Attempt any two. **10**

1. If $y = e^{ax}\cos(bx+c),\ a,b,c \in R$ then prove that

$y_n = r^n e^{ax}\cos(bx + c + n\alpha)$,

where $a = r\cos\alpha,\ b = r\sin\alpha,\ r = \sqrt{a^2+b^2},\ \alpha = \tan^{-1}\dfrac{b}{a}$.

2. Prove that $\log(1-x^2) = -x^2 - \dfrac{x^4}{2} - \dfrac{x^6}{3} - .....$

3. If $y = \cos^{-1}x, x \in (-1,1)$ then prove that

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0.$

2 **(a)** For $n \in N$, Obtain reduction formula **07**

$$\int \cos^n x\,dx = -\frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx.$$

From this prove that if $I_n = \int_0^{\pi/2} \cos^n x\,dx$ then $I_n = \dfrac{n-1}{n} I_{n-2}$

OR

(a) Prove that formula of length of arc is $S=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx.$ **07**

(b) Attempt any two. **08**

1. $\lim_{n\to\infty}\left[\frac{1}{1^2+n^2}+\frac{2}{2^2+n^2}+\frac{3}{3^2+n^2}+\cdots\cdots+\frac{n}{n^2+n^2}\right].$

2. Evaluate $\int_0^1 x^4\left(2-x^2\right)^{\frac{3}{2}}\,dx.$

3. Find the length of cycloid equation is
$x=a(\theta-\sin\theta),\ y=a(1-\cos\theta).$

3 **(a)** Vectors $\bar{a},\bar{b},\bar{c}$ are coplanar if and only if $\left[\bar{a}\ \ \bar{b}\ \ \bar{c}\right]=0.$ **06**

OR

(a) Obtain the polar equation of a straight line passing through the points (r_1,θ_1) and (r_2,θ_2). **06**

(b) Attempt any two. **10**

1. Prove that $\nabla^2 f(r)=f''(r)+\frac{2}{r}f'(r)$, where $r=|\bar{r}|$.

2. Find the centre and radius of circle
$x^2+y^2+z^2-2y+2z-23=0,\ x+2y-2z+5=0$

3. Find out reciprocal vector set of the set $\{(4,1,2),(2,-1,1),(-1,-1,1)\}$.

4 **(a)** If the plane $lx+my+nz=p$ touches the sphere **07**
$x^2+y^2+z^2=a^2$
obtain the condition and point of contact.

OR

(a) Obtain equation of the tangent plane to a sphere **07**
$x^2+y^2+z^2=a^2$ at point $P(\alpha,\beta,\gamma)$.

(b) Attempt any two. **08**

1. Find the value of k if the plane $2x-y+3z=k$ touches to the sphere $x^2+y^2+z^2=7$

2. Find the equation of circle through the point $(0,0,0),(0,0,1)$, $(2,0,0)$ and $(0,3,0)$.

3. Find the equation of the sphere passing through the circle $x^2+y^2+z^2=9, 2x+3y+4z=5$ and point $(1,2,3)$.

5 Attempt any two.

1. The functions $f(x)=x^3$, & $g(x)=x^2$, $x>1$ then prove that $\frac{3}{2}<\frac{x^3-1}{x^2-1}<\frac{3}{2}x.$ **08**

2. If $F(x,y,z)=x^2\,\bar{i}+y^2\,\bar{j}+z^2\,\bar{k}$ then prove that $curl\,F=0$.

3. Find the equation of a cylinder whose axis is $\frac{x}{1}=\frac{y}{-1}=\frac{z}{-1}$ and the guiding curve is $2x^2+3y^2=1, z=0$.