AAH-7312 Seat No. ________

M. Sc. (Sem. II) Examination

April / May - 2018

Mathematics : Paper - MTHP - 3

(Complex Analysis)

Time : 3 Hours] [Total Marks : 90

Instructions: 1. all questions are compulsory.
2. Standard notations and conventions are followed.

1 Answer the following (Any Three) 18

(1) State and Prove Cauchy – Riemann Equations.

(2) State and Prove De Moivre's Theorem.

(3) Find the harmonic conjugate of $u(x, y) = y^3 - 3x^2 y$ using Milne Thomson Method.

(4) Find all the values of $(1)^{\frac{1}{8}}$.

(5) Show that the following $f(z)$ satisfy CR equations but not differentiable at origin?

$$f(z) = \begin{cases} \dfrac{\bar{z}^2}{z} & \text{when } z \neq 0 \\ 0 & \text{when } z = 0 \end{cases}$$

2 Answer the following (Any Three) 18

(1) State and Prove Cauchy Integral Theorem.

(2) State and Prove Liouville's Theorem.

(3) Evaluate $\int_C \operatorname{Re}(z^2)dz$ where C is the boundary of the square with vertices $0,\ i, 1+i, 1$ in the clockwise direction.

(4) Evaluate $\int_C z^2 dz$ where C is a parabola $y = x^2$ from (1, 1) to (2, 4).

(5) Evaluate $\int_C (5z^2+4z+1)dz$ where C: |z|=1.

3 Answer the following (Any Three) **18**

(1) State and Prove Cauchy Integral Formula.

(2) Evaluate $\int_C \frac{dz}{z(z-2)^3}$ where C: |z|=5.

(3) Evaluate $\int_C \frac{e^z dz}{z(1-z)^3}$ where C: |z|=2.

(4) Find Maclaurin's series of $f(z) = e^z \cosh z$.

(5) For infinite series $\sum_{n=1}^{\infty} z_n$, where $z_n = x_n + iy_n$, and $z = x + iy$. Then $\lim_{n\to\infty} z_n = z$ if and only if $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$.

4 Answer the following (Any Two) **18**

(1) State and Prove Cauchy Residue Theorem.

(2) Find Laurent series of $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in $0 < |z+1| < 3$.

(3) Using Cauchy residue theorem evaluate $\int_C \frac{dz}{(z^2+1)^2}$ where $C: |z+i| = 1$.

(4) Using Cauchy residue theorem evaluate $\int_C \tan z\, dz$ where $C: |z| = 2$.

5 Answer the following (Any Six) **18**

(1) Find modulus and argument of $(\sqrt{3}-i)^6$.

(2) $\lim_{z\to i}\frac{z^2+1}{z-i}$

(3) Expand $\sin 4\theta$ in powers of $\sin\theta$ and $\cos\theta$.

(4) State Morera's Theorem and Fundamental Theorem of Algebra.

(5) Find real and imaginary part of $(1+\sqrt{3}i)^i$.

(6) State and prove relation between $\sin x$ and $\sinh x$.

(7) Show that $f(z) = Re\ (z)$ is continuous but not differentiable.

(8) Find the residue at $z = 0$ of $f(z) = z\cos(1/z)$.

(9) Find the Laurent series of $\frac{1+2z}{z^2+z^3}$ in $0 < |z| < 1$.
