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AAJ-7321-7322 Seat No._____

M. Sc. (Sem. II) Examination

April / May - 2018

(1) : Mathematics : MTHE C - 6

(Statistical Methods)

(2) Mathematics : Paper - MTHE A - 3

(Number Theory)

Time : Hours] [Total Marks : 70

(1) : Mathematics : MTHE C - 6

(Statistical Methods)

Instructions : (1) All questions are compulsory.

(2) Follow standard notations and conventions.

(3) Each question carried 14 marks.

1 Attempt any three of the following :

(a) Calculate coefficient of correlation between X and Y series from the following data and calculate its probable error also :

X : 78 79 96 69 59 79 68 61

Y : 125 137 56 112 107 136 123 108

(take 69 as working mean for 12 and 112 for that Y)

(b) Explain: types of correlation.

(c) Consider a sample with data values of 27, 25, 20, 15, 30, 34, 28, and 25.

Compute the range, interquartile range, variance, and standard deviation.

(d) Define arithmetic mean and geometric mean then Calculate arithmetic and geometric mean of the following distribution

X :	2	3	4	5	6	7	8
F :	2	4	6	2	3	2	1

2 Attempt any **three** of the following :

(a) How many ways can three items be selected from a group of six items? Use the letters A, B,C, D, E, and F to identify the items, and list each of the different combinations of three items.

(b) A committee of four has to be formed from among 3 mathematics, 4 chemistry, 2 physics and 1 life science then

(i) What is the probability there each of four is represent in the committee.

(ii) What is the probability that the committee consist life science at least one mathematics.

(c) State and prove Rule for the Inverse Probability.

(d) Consider the experiment of tossing a coin three times.

(i) Develop a tree diagram for the experiment.

(ii) List the experimental outcomes.

(iii) What is the probability for each experimental outcome ?

3 Attempt any **three** of the following :

(a) How many Types of Sampling? Explain Any Two.

(b) A random sample of 500 pineapples was taken from a large consignment and 65 of them were found to be bad. Show that the standard error of the proportion of bad ones in a sample of this size is 0.015 and deduce that. the percentage of bad pineapples in the consignment almost certainly lies between 8.5 and 17.5.

(c) A 95% confidence interval for a population, mean was reported to be 152 to 160. If $\sigma = 15$ 15, what sample size was used in this study?

(d) How large a sample should be selected to provide a 95% confidence interval with a margin of error of 10? Assume that the population standard deviation is 40.

4 Attempt any **three** of the following :

(a) Explain it: Simple Linear Regression Model And last square Method.

(b) The regression equations of two variables are $5y = 9x - 22$ and $20x = 9y + 350$ then find means of x, y and the value of r.

(c) Derive formula for Angle between the Regression Lines.

(d) Write the equation of lines of regression. Find the equation of regression lines.

If $\bar{x} = 30.4, \bar{y} = 26.5, \sigma_x = 6.4, \sigma_Y = 8$ and $r = 0.56$

5 Attempt any **four** of the following :

(a) What is the Different between passion and exponential distribution ?

(b) Explain :

(i) Independent events

(ii) Exhaustive events.

(c) Two Cards are drawn from a well shuffled pack of 52 cards. Find the probability that both are kings.

(d) Three groups of children contain respectively 3 girls and I boy, 2 girls and 2 boys, 1 girl and 3 boys. No child is selected at random from each group. Show that the chance that the three selected consist of I girl and 2 boys is 13/32.

(e) A bag contains 4 white, 5 red and 6 green balls. Three balls are drawn at random. What is the chance that a white, a red and a green ball is drawn?

(2) Mathematics : Paper - MTHE A - 3

(Number Theory)

Instructions: 1. All questions are compulsory.

2. Standard notations and conventions are followes.

1 Answer any **TWO** of the following: **14**

(1) Define Möbius function μ. State and prove Möbius inversion formula.

(2) Show that there is no end to the sequence of primes $2, 3, 5, 7, 11, 13, 17 \ldots$.
Also find the upper bound for the n^{th} prime of this sequence.

(3) Define Euler's totient function $\phi(n)$. For $n > 2$, prove that $\phi(n)$ is always an even integer.

2 Answer any **TWO** of the following: **14**

(1) State and prove Chinese Remainder Theorem.

(2) Find the last two digits of 9^{9^9}.

(3) Prove that any integer of the form $2^k; k \geq 3$ do not have a primitive root.

3 Answer any **TWO** of the following: **14**

(1) State and prove the Quadratic Reciprocity law.

(2) If p is an odd prime then prove that

$$\left(\frac{2}{p}\right) = \begin{cases} 1 & ; \text{if } p \equiv \pm 1 \ (mod\ 8) \\ -1 & ; \text{if } p \equiv \pm 3 \ (mod\ 8) \end{cases}.$$

(3) Prove the following:

(a) -1 is a quadratic residue of p if and only if p is of the form $4k + 1$.

(b) 3 is a quadratic residue of p if and only if p is of the form $12k \pm 1$.

4 Answer any **TWO** of the following: **14**

(1) Determine all the positive solutions of the Diophantine equation $172x + 20y = 1000$.

(2) If x, y, z are positive primitive Pythagorean triplets and x is even then derive an expression which gives all the values of x, y, z.

(3) If a positive integer n is expressed as $n = N^2m$, where m is square-free then show that n can be represented as the sum of two squares if and only if m contains no prime factors of the form $4k + 3$.

5 Answer the following: **14**

(1) If $7 \nmid a$, prove that either $a^3 + 1$ *or* $a^3 - 1$ is divisible by 7.

(2) Find the value of $\tau(360)$ and $\sigma(360)$.

(3) If n is an odd integer then prove that $\phi(2n) = \phi(n)$.

(4) Find the remainder when 9^{1032} is divided by 13.

(5) Obtain a primitive root of 25.

(6) Check which of the quadratic congruence $x^2 \equiv 196(mod\ 1357)$ has a solution.

(7) Express 2431 as the product of primes.
