AT-1917-18 Seat No. ________

B. Sc. (Sem. VI) Examination

March / April – 2018

Mathematics : Paper No. CC -MATH - 603

(A) General Topology

(B) Number Theory

Time : **3** Hours] [Total Marks : **70**

(A) General Topology

Instructions : (1) All questions are compulsory; there are five questions.

(2) Figures to the right indicate marks of the corresponding question.

1 (a) Define topological space. **6**

Prove that :

A subset O of a topological space X is open

iff

O is a neighborhood of each of its points.

(b) Define closure of a subset of a topological space. **6**

Prove that :

A subset $\mathbb{A}$ of a topological space X is closed.

iff

$\mathbb{A} = \overline{\mathbb{A}}$

(c) Show that : **6**

The intersection $T_1 \cap T_2$ of any two topologies T_1 and T_2 on X is also a topology on X.

OR

1 (a) Define Hausdorff topological space. **6**

Prove that :

Subspace of a Hausdorff topological space is also a Hausdorff space.

(b) If $\mathbb{A}$ and $\mathbb{B}$ are subsets of a topological space X; **6**

then prove that $\overline{\overline{\mathbb{A}}} = \mathbb{A}$ and $\overline{\mathbb{A} \cup \mathbb{B}} = \overline{\mathbb{A}} \cup \overline{\mathbb{B}}$.

(c) Consider **6**

$$T = \{\phi, X, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$$

be a topology on $X = \{a, b, c, d, e\}$.

Then find out $\overline{\{b\}}$ and $\overline{\{b, d\}}$.

2 (a) Define interior of a subset of a topological space. **6**

Let $\mathbb{A}$ be a subset of a topological space X.

Prove that :

$Int.(\mathbb{A})$ is the largest open set contained in $\mathbb{A}$.

(b) Define continuous function from one topological space X to another topological space Y. **6**

Prove that :

$f(X, T) \to (Y, T')$ is continuous

iff

for each subset $\mathbb{A}$ of X; $f(\overline{\mathbb{A}}) \subset \overline{f(\mathbb{A})}$.

(c) Let $\mathbb{A}$ be a non-empty proper susbet of an indiscrete topological space X. **6**

Then find out $Int.(\mathbb{A})$ and $Bdry(\mathbb{A})$.

OR

2 (a) Defind homeomorphic topological spaces. **6**

Prove that :

A necessary and sufficient condition that two topological spaces (X, T) and (Y, T') be homeomorphic is that there exists a function $f : X \to Y$ s.t.

(1) f is one-one

(2) f is onto

(3) A subset O of X is open iff $f(O)$ is open.

(b) Prove that : **6**

A function $f : (X, T) \to (Y, T')$ is continuous

iff

for each open subset O of Y; $f^{-1}(O)$ is an open susbet of X.

(b) Show that : **6**

The identity function $I : (X, T) \to (X, T')$ is continuous

iff

T is finer than T'.

3 (a) Define subspace of a topological space. 6

Prove that :

If Y is a subspace of X and if

$$T' = \{O' \subset Y / O' = O \cap Y;\ \text{where } O \text{ open in } X\};$$

then T' is a topology on Y. Where $Y \neq \phi$.

(b) State and prove : Intermediate-Value Theorem. 6

(c) Let $\mathbb{A} = \{b, d, e\}$ be a subset of 6

$X = \{a, b, c, d, e\}$ and

$T = \{\phi, X, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$

be a topology on X. Is $\mathbb{A}$ connected subset of X ? Why ?

OR

3 (a) State and prove : Fixed-Point Theorem. 6

(b) Define component of a point of a topological space. 6

Prove that :

In a topological space X, if $b \in C_{mp}(a)$, then

$C_{mp}(b) = C_{mp}(a)$.

(c) Determine the components of a discrete space. 6

4 Attempt any **two** : **8**

(1) Prove that the closure of a connected set is connected.

(2) Prove that for subset $\mathbb{A}$; $Bdry(\mathbb{A})$ is closed.

(3) Given a subset $\mathbb{A}$ of a topological space and a point $x \notin \overline{\mathbb{A}}$; then show that $x \notin F$ for some closed set F containing $\mathbb{A}$.

5 Attempt any **two** : **8**

(a) Let $T = \{\mathbb{R}\} \cup \{\phi\} \cup \left\{ A_q = (q, \infty) / q \in Q \right\}$.

Is T topology on $\mathbb{R}$?

(b) Show that $\overline{\mathbb{A}} = \mathbb{A} \cup Bdry(\mathbb{A})$; where $\mathbb{A}$ is a subset of a topological space.

(c) Let $X = \{1, 2, 3\}$ with a topology $T = \{\phi, X, \{1\}, \{2\}, \{1, 2\}\}$. For $3 \in X$; find Comp.(3).

(B) Number Theory

Instructions : (1) All questions are compulsory.

(2) Figures to the right indicates the marks of the corresponding question.

1 (A) Prove Binomial theorem by induction **6**

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n}b^n$$

OR

(A) State and prove the Division Algorithm Theorem.

(B) Attempt any **three** : **12**

(1) Find the integers x and y satisfying gcd $(2017, 1969) = 2017x + 1969y$ by Euclidean Algorithm.

(2) Show that $8 | 5^{2n} + 7$ by Mathematical Induction.

(3) If g.c.d. $(a, b) = 1$ then prove that g.c.d. $(2a+b, a+2b) = 1 \; or \; 3$.

(4) Solve the Diophantine equation $54x + 21y = 906$. And also find out the positive solution.

2 (A) Prove that there are infinitely many prime of the form $4k+3$. **6**

OR

(A) Let $n>0$ by fixed and a,b,c be any integers

(i) if $a \equiv b(\bmod n), b \equiv c(\bmod n) \Rightarrow a \equiv c(\bmod n)$

(ii) if $a \equiv b(\bmod n) \Rightarrow a^2 \equiv b^2(\bmod n)$

(B) Attempt any **three** : **12**

(1) Solve the linear Congruence $17x \equiv 3(\bmod 210)$.

(2) If p_n is the n^{th} prime number, then $p_n \le 2^{2^{n-1}}$ by mathematical induction.

(3) Prove any prime of the form $3n+1$ is also of the form $6m+1$.

(4) Give and example to show that $a^2 \equiv b^2(\bmod n)$ need not imply that $a \equiv b(\bmod n)$.

3 (A) State and prove Wilson's Theorem. **6**

OR

(A) Prove that the function ϕ is multiplicative.

(B) Attempt any **three** : **12**

(1) By Euler's Theorem, find the remainder when 5^{38} is divisible by 11.

(2) For a prime p, prove that

$$p / a^p + (p-1)!a \text{ and } p / a + (p-1)!a^p .$$

(3) Prove that

$$1^{p-1} + 2^{p-1} + \ldots + (p-1)^{p-1} \equiv -1 (\bmod p).$$

(4) If p and q are distinct primes such that $a^p \equiv a(\bmod q)$ and $a^q \equiv a(\bmod p)$ then $a^{pq} \equiv a(\bmod pq)$

4 Attempt any **four** : 16

(1) Find the remainder when $1^5 + 2^5 + 3^5 + \ldots + 100^5$ is divisible by 5.

(2) If c / ab and $(c, a) = 1$, then c / b.

(3) If $a \equiv b(\bmod m)$ and $c \equiv d(\bmod m)$ then $ac \equiv bd(\bmod m)$.

(4) If $n > 2$ then $\phi(n)$ is even.

(5) Use the Sieve of Eratosthenes find the all primes $p \leq 100$.

(6) $\sqrt{p}$ is irrational for any prime p.

(7) Define g.c.d., if a / c and b / c, with gcd $(a, b) = 1$, then ab / c.