AT-1809

Seat No._____

B. Sc. (Sem. IV) Examination

March / April – 2018

CC MATH-401 : Mathematics

(Advance Calculus)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) All four questions are compulsory.

(2) Figure to the right indicate marks of the question.

1 (a) Obtain the formula for radius of curvature of the curve $y = f(x)$. **5**

OR

(a) Prove the relation $\beta(m, n) = \dfrac{\lceil m \lceil n}{\lceil m+n}$.

(b) Attempt any three : **15**

(1) Find the radius of curvature of $x = m\cos\theta + n\sin\theta,\ y = m\sin\theta - n\cos\theta$.

(2) Find the double points of curve $x^3 + y^3 - 12x - 27y + 70 = 0$ and explain their type.

(3) Prove that :

$$\int_0^1 \frac{x^2\,dx}{\left(1-x^4\right)^{1/2}} \cdot \int_0^1 \frac{dx}{\left(1-x^4\right)^{1/2}} = \frac{\pi}{4}$$

(4) Prove that :

$$\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$$

2 (a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \left(x^2+y^2\right) dx\,dy$ 5

by transforming into polar co-ordinate.

OR

(a) Find the volume of $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

(b) Attempt any three : 15

(1) Evaluate

$$\int_0^{\pi/2} \int_{-1}^{4} \left(x\sin y - ye^x\right) dy\,dx$$

(2) Find $\iint \left(x^2+y^2\right) dx\,dy$ over the region bounded by $x=1,\ x=2,\ y=1,\ y=x^2$.

(3) Change the order of integration

$$\int_0^3 \int_{4y/3}^{\sqrt{25-y^2}} f(x, y)\, dy \cdot dx$$

(4) Evaluate : $\iint_s r^2 \sin\theta\, ds$, where s is the area of cardioids above the initial line.

3 (a) State and prove Green's theorem. **6**

OR

(a) State and prove Stoke's theorem.

(b) Attempt any three : **15**

(1) Verify Green's theorem for

$\int_c \left(3x^2 - 8y^2\right) dx + (4y - 6xy)\, dy$, where c is the boundary of region bounded by $y^2 = x$ and $x^2 = y$.

(2) Evaluate $\iint_s f \cdot n\, ds$, where

$f = \left(x + y^2, -2x, 2yz\right)$ and surface s is the plane $2x + y + 2z = 6$ in the first octant.

(3) Verify $div\,(curl\ f) = 0$, if

$f = \left(x^2 y, -2xz, 2yz\right)$.

(4) Prove that

(1) $div.(\varphi f) = \varphi \quad div.f + f\,(grad\ \varphi)$

4 Attempt any three : 9

(1) Discuss the type of double point (0,0) of

$$x^3 + x^2 - 4y^2 = 0$$

(2) Find the radius of curvature of the curve $y = \log x$.

(3) Evaluate : $\int_0^1 \sqrt[3]{x \log\left(\frac{1}{x}\right)}\, dx$

(4) Evaluate : $\int_0^1 \int_0^3 \left(x^2 + y^2\right)\, dx\, dy$.
