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AAI-5704 Seat No._____

B. Sc. (Sem. II) Examination

April/May - 2018

Mathematics : CCMAT-122

Time : 3 Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.
(2) Figures to the right indicates the marks of the corresponding question.

1 (a) State and prove De'Moiver's theorem. 7

OR

(a) Obtain expansion of $\sin n\theta, \cos n\theta, \tan n\theta$ in terms of $\sin\theta, \cos\theta$ and $\tan\theta$ respectively.

(b) Attempt any two : 8

(1) If $p = cis\,\theta, q = cis\,\phi$ then prove that

$$\frac{p-q}{p+q} = i\tan\left(\frac{\theta-\phi}{2}\right)$$

(2) Prove that :

$$\cos^8\theta = \frac{1}{128}[\cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35]$$

(3) Prove that :

$$2(1+\cos 8\theta) = \left(x^4 - 4x^2 + 2\right)^2, \text{ where } x = 2\cos\theta \text{ using De-Moivre's theorem.}$$

2 (a) Prove that : 7

$$\tan^{-1} h(z) = \frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$$

OR

(a) State and prove the "Cauchy's Root Test".

(b) Attempt any two : 8

(1) If $u = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$ then prove that

$$\tanh\left(\frac{u}{2}\right) = \tan\frac{\theta}{2}.$$

(2) Prove that $Log\left(\frac{3-i}{3+i}\right) = 2i\left(n\pi - \tan^{-1}\frac{1}{3}\right)$

(3) $\sum\left(1-\frac{1}{n}\right)^{n^2}$ converges ? Why ?

3 (a) Define linear differential equation and write the method of solving it. 7

OR

(a) Prove that $\frac{1}{f(D)}\left(e^{ax}\cdot V\right) = e^{ax}\cdot\frac{1}{f(D+a)}V$.

Where V is a function of x.

(b) Solve any two : 8

(1) $\frac{dy}{dx}+\frac{2}{x}\cdot y=\frac{y^3}{x^3}$

(2) $x\cdot p^3-2yp^2+x^2=0;\ p=\frac{dy}{dx}$

(3) $\left(D^2+1\right)y=5x^2$.

4 (a) Prove that every square matrix can be represent unique as a sum of a symmetric and a skew symmetric matrix. 7

OR

(a) Let $A=\left[a_{ij}\right]_n$ be a square matrix with order n then prove that, $A(adjA)=(adjA)A=|A|I_n$.

(b) Attempt any two : 8

(1) Find the rank of matrix

$$A=\begin{bmatrix}0 & 3 & -2 & 4\\ -1 & 4 & -3 & 3\\ 1 & -7 & 5 & -7\end{bmatrix}$$ using Row-Reduction method.

(2) Given an example of a Hermitian and a skew-Hermitian matrix of order 3.

(3) Find A^{-1} for matrix $A=\begin{bmatrix}2 & 1 & 4\\ 4 & 3 & 1\\ 1 & 2 & 4\end{bmatrix}$ using Row-Reduction method.

5 Attempt any **five** : **10**

(1) Solve by De-Moivre's theorem : $x^5+1=0$.

(2) If $Z=\left(1+i\sqrt{3}\right)^{13}$ then find $|z|$ and principal Arg. of Z.

(3) If $y=\log\tan x$ then prove that,

$$\sinh(ny)=\frac{1}{2}\left[\tan^n x-\cot^n x\right].$$

(4) Solve : $\sin z=\cosh(4)$.

(5) Solve : $\left(D^2-3D+2\right)y=e^{7x}$.

(6) Solve : $y=px+p^3+\log p+\sin p$, where $p=\frac{dy}{dx}$.

(7) Find the radius of convergence of the series

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n\cdot 2^n}x^n.$$

(8) If A and B are symmetric matrices of same order then prove that $AB - BA$ is skew symmetric matrix.
