P.S.SCIENCE & H.D.PATEL ARTS COLLEGE, KADI

INTERNAL EXAMINATION

B.Sc. Sem -VI | Marks 40

07/03/2018 | Mathematics | Time: 1.45 to 3.45

CC-MATH- 602

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1. (A) Attempt any two.

(i) Let $f_1, f_2, f_3, \ldots, f_k$ be the real function on a metric space X and Let $\overline{f}$ be the mapping of X into R^k defined by $\overline{f}(x) = (f_1(x), f_2(x), \ldots\ldots, f_k(x)), (x \in X)$ then $\overline{f}$ is continuous if and only if each f_i is continuous where $i = 1, 2, 3, \ldots, k$.

(ii) Let f be monotonically increasing on (a,b) then prove that $f(x+)$ & $f(x-)$ exists at every point of x of (a,b).

(iii) Show that continuous image of compact is compact.

(iv) If f is a real continuous function on $[a,b]$ which is differentiable in (a,b), then there is a point $x \in (a,b)$ at which

$$f(b) - f(a) = (b-a)f'(x)$$

(B) Attempt any one.

(i) Define

$$f(x) = x+1 \ , 0 \le x < 1$$
$$= 2x+3 \ , 1 \le x \le 2$$

Discuss the continuity at $x = 1$

(ii) Evaluate

$$\lim_{x \to 0} \frac{x - \tan x}{x^3} \text{ and } \lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x}}$$

2. (A) Attempt any two.

(i) State and prove necessary & sufficient condition for $f \in R(\alpha)$.

(ii) $f \in R(\alpha_1)$ & $f \in R(\alpha_2)$ then prove

(a) $f \in R(\alpha_1 + \alpha_2)$ (b) $\int_a^b f \, d(\alpha_1 + \alpha_2) = \int_a^b f \, d\alpha_1 + \int_a^b f \, d\alpha_2$

(iii) State & prove the Fundamental theorem of calculus.

(B) Attempt any one.

(i) Suppose α increases on $[a, b]$, $a \le x_0 \le b$, α is continuous at x_0, $f(x_0) = 1$ *and* $f(x) = 0$ *if* $x \ne x_0$ then prove that $f \in R(\alpha)$ and $\int_a^b f \, d\alpha = 0$

(ii) If $f(x) = \int_x^{x+1} sin(e^t)dt$ then prove that, $e^x \cdot |f(X)| \le 2$.

3. (A) Attempt any two.

(i) Prove that the sequence $\{f_n\}$ of functions defined on E converges uniformly on E if and only if for every $\varepsilon > 0$, $\exists$ a positive integer N such that $m, n \ge N \Rightarrow |f_n(x) - f_m(x)| < \varepsilon, \forall x \in E$

(ii) Let α be monotonically increasing on $[a,b]$. Suppose $f_n \in R(\alpha)$ on $[a,b]$, for $n = 1, 2, 3, \ldots$ and suppose $f_n \to f$ uniformly on $[a,b]$ then prove that $f \in R(\alpha)$ on $[a,b]$ and $\int_a^b f \, d\alpha = \lim_{n\to\infty} \int_a^b f_n \, d\alpha$.

(iii) If K is a compact metric space if $f_n \in C(K)$, $n = 1, 2, 3, \ldots$ and if $\{f_n\}$ is point wise bounded and equi -continuous on K then prove that $\{f_n\}$ is uniformly bounded on K

(B) Attempt any one·

(i) If $f_n(x) = \dfrac{1}{1+nx}$, $x \in (0,1)$ then show that $f_n(x) \to 0$ point wise on $(0,1)$ but the convergence is not uniform.

(ii) Give an example of a sequence of functions for which

$$\lim_{n\to\infty}\left[\int_0^1 f_n(x)\,dx\right] \ne \int_0^1\left[\lim_{n\to\infty} f_n(x)\right]dx$$
