Pramukh Swami Science & H D Patel Arts College, Kadi
Internal Examination, Oct-2017,
B.Sc. Semester- VI (Mathematics)
CC MATH-601 Ring theory

Date:06/03/2018
Time: 01:45 to 03:45 **Total Marks: 40**

1. (a) Define characteristic of a ring and Show that, the characteristic of an integral domain is either a prime number or zero. **[06]**

OR

Prove that, A non-zero element m of a ring $(Z_n, +_n, \times_n)$ is a zero divisor if and only if m and n are not relatively prime.

(b) Attempt any three of the following: **[09]**

1) Prove that, If p is the characteristic of an integral domain D then $(a+b)^p = a^p+b^p$, for $a, b \in D$.
2) Prove that, a field has no proper ideal.
3) Obtain all the zero divisors of ring $(Z_{12}, +_{12}, \times_{12})$.
4) Give an example of the following.
   (1) Commutative ring which is not an integral domain.
   (2) Division ring which is not a field
   (3) Integral domain which is not a field.

2. a) State and prove Eisenstein Criterion. **[07]**

OR

a) Define primitive polynomial and prove that, the product of two primitive polynomials in Z[x] is also a primitive polynomial in Z[x]

(b) Attempt any two of the following: **[06]**

1) Define a cyclotmic polynomial and show that, it is an irreducible over **Q**.
2) Obtain all rational roots of $f(x) = 4x^5 + x^3 + x^2 - 3x + 1$ and its factorization.

3) Find the g.c.d of $f(x)=6x^3+5x^2-2x+25$ and $g(x)=2x^2-3x+5 \in R[x]$ and express f(x) in the form $a(x)f(x)+b(x)g(x)$.

(a) An ideal $I=\langle p \rangle$ is a maximal ideal of ring $\langle Z, +, . \rangle$ if and only if p is prime.. [06]

OR

Let (R;+;.) be a ring with unity. Prove that the mapping $\phi:(Z;+;.)\rightarrow(R;+;.)$ defined by $\phi(n)=n.1, n\in Z$ is a homomorphism with $K_\phi=\langle m \rangle$, where m is the characteristic of R.

(b) Attempt any two of the following: [06]

1) Show that ideal $I=\langle x^3-x-1 \rangle$ is a maximal ideal in $Z_3[x]$.
2) Show that, $I = \{0\}$ is a prime ideal but not maximal ideal in $(Z, +, \cdot)$
3) A homomorphism defined on the ring (Z, +, .) is either zero homomorphism or identity mapping.

BEST OF LUCK