KS-1643 Seat No.__________

M. Sc. (Sem. I) Examination

November / December – 2017

Mathematics : MTHE-2

(Techniques of Differential Equations)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Standard notations and conventions are followed.

1 Attempt any three : **15**

(a) Solve the equation

$$\left(2xyz + z^2\right) dx + x^2 z\, dy + (xz + 1)\, dz = 0.$$

(b) Solve the equation

$$2xz(y - z)\, dx - z\left(x^2 + 2z\right) dy + y\left(x^2 + 2y\right) dz = 0$$

by Natani's method.

(c) A necessary there exist between two functions $u(x, y)$ and $v(x, y)$ a relation $F(u, v) = 0$, not involving x and y explicity is that $\dfrac{\partial(u, v)}{\partial(x, y)} = 0$.

(d) Using one separable method solve the equation

$$x\left(y^2-a^2\right)dx+y\left(x^2-z^2\right)dy-z\left(y^2-a^2\right)dz=0.$$

2 Attempt any three : **15**

(a) Derive formula of a Jacobi method.

(b) If $U(x,y,z)=C_1$, $V(x,y,z)=C_2$ is a solution of $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$ then prove that $F(U,V)=0$ is general solution of $P_p+Q_q=R$.

(c) Find the general integral equation $(x-y)p+(y-x-z)q=z$ and the particular solution through the circle $z=1$ and $x^2+y^2=1$.

(d) Use Charpit's method find the solution of the equation $xp+3yq=2\left(z-x^2q^2\right)$.

3 Attempt any two : **14**

(a) Solve the equation $p+r+s=1$.

(b) Solve $r+4s+t+rt-s^2=2$ by Monge's method.

(c) Find a surface passing through the two lines $z=x=0$, $z-1=x-y=0$ satisfying $r-4s+4t=0$.

4 Attempt any two : **14**

(a) State and solve Dirichlet problem in rectangle.

(b) Show that the surfaces $x^2 + y^2 + z^2 = c \cdot x^{2/3}$.

(c) Using the method of separation of variables,

solve $\dfrac{\partial u}{\partial x} = 2\dfrac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$.

5 Attempt any three : **12**

(a) Solve the equation

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-z)}.$$

(b) Find the complete integral of the equation

$p_1^2 + p_2^2 + p_3 = 1$.

(c) Obtain differential equation of

$u = f(x + ct) + g(x - ct)$ where f and g are arbitrary function.

(d) Find the complete integral of the equations

$p^2 z^2 + q^2 = 1$.

(e) Prove that a necessary condition for the existence of a solution of the interior Neumann problem is that the integral of f over the boundary S should variable i.e. $\int_g f(p)\, ds = 0$.
