



**KU-1673**

Seat No. \_\_\_\_\_

**M. Sc. (Sem. I) Examination**

**November / December - 2017**

**Mathematics : Paper-MCB-I**

**(Special Function)**

Time : 2 Hours]

[Total Marks : 35

**Instructions :**

- (1) All questions are compulsory.
- (2) Standard notations and conventions are followed.
- (3) Each question carry equal marks.

1 Find the power series of the equations  $y'' - xy' + 2py = 0$ . Where  $p$  is non-Negative integer. 7

**OR**

1 (a) Find the power series Solution on of  $y'' - xy' = 0$  near  $x_0 = 0$ . 4

(b) Determine the series solution of the second order differential equation  $y'' = y$ . 3

2 Determine the solution of the second order differential equation :

$$xy'' + y' + 2y = 0 \text{ with } y(1) = 1, y'(1) = 2,$$

$$y(x_0) = 1, y'(x_0) = 2.$$

**OR**

2 Show that the D.E.  $x^2y'' - 3xy' + (4x + 4)y = 0$  has only one Frobenius series. Also find the solution. 7

3 Attempt any two :

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(a) Determine the nature of Bessel's equation  
 $x^2y'' + xy' + (x^2 - p^2)y = 0$  of point  $x_0 = \infty$ .

(b) Show that  $\sin x = x \left[ \log_{a \rightarrow \infty} F \left( a, a, \frac{3}{2}, \frac{-x^2}{4a^2} \right) \right]$ .

(c) Find the general solution of  $(1 - e^x)y'' + \frac{1}{2}y' + e^xy = 0$  at  $x = 0$ .

4 Prove that Generating function of Hermite is

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given by  $e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{H_n(x)}{n!} t^n$ .

OR

Prove that :

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$$\int_0^1 x J_p(\lambda_m x) J_p(\lambda_n x) = \begin{cases} 0 & \text{if } m \neq n \\ \frac{1}{2} J_{p+1}(\lambda_n) & \text{if } m = n \end{cases}$$

5 Do any two :

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(i) Find Singular, ordinary, regular singular point of Equation and explain it.

$$x^3(x-1)y'' - 2(x-1)y' + 3xy = 0$$

(ii) Find power series solution of  $y' + y = 1$ .

(iii) Prove that  $J_p' - \frac{p}{x} J_p(x) = J_{p+1}(x)$ .