KR-1634 Seat No.________

M. Sc. (Sem. I) Examination

November / December – 2017

Mathematics : Paper-MTHP-2

(Algebra-I)

Time : **3** Hours] [Total Marks : **90**

Instructions : (1) All questions are compulsory.

(2) Standard notations and conventions are followed.

(3) Each questions carry equal marks.

1 Attempt any three of the following questions : **18**

(a) Show that if N and M normal subgroups of G then $\frac{NM}{M} \cong \frac{N}{M \cap N}$.

(b) State and prove Cauchy's theorem for finite abelian group.

(c) Prove that if ϕ is homomorphism of G onto $\overline{G}$ with kernel k then $G / k \cong \overline{G}$.

(d) Show that a subgroup of a solvable group is solvable.

2 Attempt any three of the following questions : **18**

(a) State and prove Sylow's theorem of finite abelian group.

(b) Show that the number of conjugate classses in the symmetric group S_n of degree n is equal to the number of partition of n.

(c) State and prove first part of Sylow's theorem.

(d) Construct a non-abelian group of order 21.

3 Attempt any three of the following questions : **18**

(a) Prove that every Euclidean ring is an unique factorization domain.

(b) If R is a Euclidean ring show that ideal $A = \langle a_0 \rangle$ is maximal ideal in R iff a_0 is a prime element in R.

(c) Prove that a g.c.d. d of two non-zero elements a, b in a Euclidean ring R exists in R, and it can written as $d=ax+by$, where $x, y \in R$.

(d) Show that every integral domain can be imbedded in a field.

4 Attempt any three of the following questions : **18**

(a) If R is a unique factorization domain show that, the ring $R[x]$ is also a unique factorization domain.

(b) Prove that every irreducible element in a UFD is a prime element.

(c) Prove that :

(1) The polynomial $f(x) = 1 + x + x^3 + x^4$ over any field F is not irreducible.

(2) The polynomial $g(x) = 2 + 2x + x^4$ over the field $F = Q$ is irreducible.

(d) If R is an integral domain and F is its field of fractions, prove that every polynomial

$f(x) \in F[x]$ can be written as $f(x) = \frac{g(x)}{a}$

where, $a \in R$ and $g(x) \in R[x]$.

5 Answer briefly any six : **18**

(a) Show that the symmetric group S_3 is solvable group.

(b) Describe all finite abelian group of order 81.

(c) Prove that any non abelian group G of order G is isomorphic to the group S_3.

(d) Prove that group of order p^2 is abelian where p is a prime number.

(e) If $f(x)$ is any polynomial with integer co-efficient show that it can be written as $dg(x)$. Where $d = c(f)$ and $g(x)$ is primitive polynomial.

(f) Im the ring $Z[i]$, find the quotient and remainder when $20+8i$ is divided by $3+4i$.

(g) For a field F, find the set of all the units in the ring $F[x]$.

(h) Prove that an element $a \neq 0$ in a Euclidean ring R is a unit in R if and only if $d(a) = d(1)$.