KC-487 Seat No.________

B. Sc. (Sem. V) Examination

October / November – 2017

Mathematics : CC-MATH-503

(Differential Equations)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) **All** questions are **compulsory**.
(2) **Figures** to the **right** indicate the marks of corresponding question.

1 (a) Prove that **6**

$$\frac{1}{f(D)}\left[e^{ax}\cdot v\right] = e^{ax}\cdot\frac{1}{f(D+a)}v$$

where $a =$ constant, $v =$ function of x.

(b) Solve : $\left(D^3 - 5D^2 + 7D - 3\right)y = e^{2x}\cdot\cosh x$. **6**

(c) Solve : $\left(D^4 + D^3 + D^2\right)y = x^2\left(a + bx\right)$. **6**

OR

1 (a) Prove that **6**

(1) $\dfrac{1}{D^2 + a^2}\cos ax = \dfrac{x}{2a}\sin ax$

(2) $\dfrac{1}{D^2 + a^2}\sin ax = \dfrac{-x}{2a}\cos ax$

(b) Solve : $\left(D^2 + 2\right) y = x^2 \cdot e^{3x}$. 6

(c) Solve : $\left(D^2 - 1\right) y = x^2 \cdot \cos x$. 6

2 (a) If the Linear differentiable equation 6

$$P_0 \cdot y^{(n)} + P_1\, y^{(n-1)} + P_2\, y^{(n-2)} + \ldots.. + P_n \cdot y = \phi(x),$$

where $P_0, P_1, P_2 \ldots\ldots. P_n$ are function of x, is exact differential equation then prove that

$$P_n + (-1)\, P_{n-1}^{(1)} + (-1)^2 \cdot P_{n-2}^{(2)} + \ldots.. + (-1)^n\, P_0^{(n)} = 0.$$

(b) Solve : $x^2 yy_2 + \left(xy_1 - y\right)^2 - 3y^2 = 0$ 6

where $y_1 = \dfrac{dy}{dx},\ y_2 = \dfrac{d^2 y}{dx^2}$.

(c) Solve : 6

$$yy^{(2)} = \left\{y^{(1)}\right\}^2 \left[1 - y^{(1)} \cos y + yy^{(1)} \sin y\right].$$

OR

2 (a) Show that the equation 6

$$x^2 \cdot y^{(3)} + 4x \cdot y^{(2)} + \left(x^2 + 2\right) y^{(1)} + 3xy = 2$$

becomes exact on being multiplied by some power of x. Obtain its first integral.

(b) Solve : $y^{(2)} = y^3 - y$ given that $y^{(1)} = 0$ for $y = 1$. **6**

(c) Solve : **6**

$$\left\{y^{(1)}\right\}^2 - yy^{(2)} = n\left[\left\{y^{(1)}\right\}^2 + a^2\left\{y^{(2)}\right\}^2\right]^{\frac{1}{2}}.$$

3 (a) Solve : $xy^{(2)} - (x-2)\,y^{(1)} - 2y = x^3$. **6**

(b) Solve : **6**

$$y^{(2)} - 4x \cdot y^{(1)} + \left(4x^2 - 1\right)y^{(1)} + y = -3e^{x^2} \cdot \sin x$$

(By Normal Form)

(c) Solve : **6**

$$(1+x)^2 \cdot y^{(2)} + (1+x)\,y^{(1)} + y = 4\cos\left(\log(1+x)\right)$$

[By changing the independent variable]

OR

3 (a) Solve : $xy^{(2)} - (x-2)\,y^{(1)} - 2y = x^3$. **6**

(b) Solve : $xy^{(2)} - (1+x)\,y^{(1)} + y = x^2$. **6**

[By factorisation of the operator]

(c) Solve : $y^{(2)} + a^2 y = \sec ax$ **6**

[By variation of parameter]

4 Solve any four : **16**

(1) $\left(D^3 - 2D^2 - 4D + 8\right) y = 0$

(2) $\left(D^2 - 5D + 6\right) y = \sin 3x$

(3) $y^{(3)} = x - \sin x$

(4) $y^{(3)} + x \cdot y^{(3)} - y^{(2)} = 0$

(5) $y^{(2)} + 2y^{(1)} + 2y = 1 + x^2$

[By method of undetermined coefficients]

(6) $\left(D^2 - 9D + 18\right) y = e^{e^{-3x}}$.
