KB-480 Seat No. ________

B. Sc. (Sem. V) Examination

October/November – 2017

Mathematics : CCMATH-502

(Mathematical Analysis-1)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Figures to the right indicate the marks of corresponding questions.

1 (a) Show that the equation $p^2 = 2$ is not satisfied by any rational p. **5**

(b) State and prove : Archimedean property. **5**

(c) Prove : If $z, w \in ⊄$; then $|z+w| \le |z| + |w|$. **4**

OR

1 (a) Show that for any cut α on Q; $\alpha + 0^* = \alpha$. **5**

(b) State and prove : Schwarz's inequality. **5**

(c) If $\overline{a}, \overline{b} \in I\!R^k$; then find out $\overline{C} \in \mathbb{R}^K$ s.t. : **4**

$|\overline{x} - \overline{a}| = 2|\overline{x} - \overline{b}|$ iff $|\overline{x} - \overline{c}| = r$; where $\overline{x} \in \mathbb{R}^K$.

2 (a) Prove : The union of a countable family of countable sets is a countable set. 5

(b) Define metric space. 5

For $x, y \in \mathbb{R}$; define $d(x, y) = \left|x^2 - y^2\right|$ and $d'(x, y) = |x - 2y|$.

Determine, for each of these, whether it is a metric or not.

(c) Let X be a metric space. 4

Prove : A subset E of X is an open set in X iff E^c is a closed set in X.

OR

2 (a) Let $E \neq \phi$; $E \subset \mathbb{R}$ and E is bounded above. 5

Then prove : $y \in \overline{E}$.

(b) Prove : Compact subsets of metric spaces are closed. 5

(c) Show that Cantor set is a perfect set. 4

3 (a) Prove : If X be a metric space and if p is a limit point of a susbet E of X; then there is a sequence $\{p_n\}$ in E s.t. $\lim_{n \to \infty} p_n = p$. 5

(b) Prove : If X is a Compact metric space and if $\{p_n\}$ is a Cauchy sequence in X; then sequence $\{p_n\}$ converges to some point of X. 5

(c) Prove : If $p \in \mathbb{R}^{+}$ and $\alpha \in \mathbb{R}$; then **4**

$$\lim_{n \to \infty} \frac{n^{\alpha}}{(1+p)^{n}} = 0.$$

OR

3 (a) State and prove : Cauchy criterian for a series. **5**

(b) Prove : $\sum \frac{1}{n^p} = \begin{cases} converges, \ if \ \ p > 1 \\ diverges, \ if \ \ p \le 1 \end{cases}$. **5**

(c) State and prove : Cauchy's root test. **4**

4 Attempt any two :

(1) Show that Q is dense in $\mathbb{R}$. **5**

(2) Show that every neighborhood of a point in a metric space is an open set. **5**

(3) Show that $\lim_{n \to \infty} \sqrt[n]{n} = 1$. **4**

5 Attempt any two :

(1) Show that O* is a cut on Q. **5**

(2) Show that closed balls are convex set in $\mathbb{R}^{K}$. **5**

(3) Discuss the convergence of a power series **4**

$$\sum_{n=0}^{\infty} \frac{z^{n}}{n!}.$$