KC-401 Seat No.________

B. Sc. (Sem. III) Examination

October / November – 2017

Mathematics : CC-MATH-301

(Calculus & Linear Algebra)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) **All** questions are **compulsory**.
(2) **Figures** to the **right** indicate the marks of the corresponding question.

1 (a) State and prove Schwartz's theorem. **8**

OR

(a) Suppose $Z = f(x, y)$ is defined on a nonempty set $E \subset R^2$ and f_x, f_y exist and continuous at (x, y) then prove that f is differentiable at $(x, y) \in E$. **8**

(b) Attempt any **two** : **10**

(1) If $f(x, y) = \frac{x^3 + y^3}{x - y}, \quad x - y \neq 0$

$= 0 \quad , \quad x - y = 0$

Prove that $f_x(0, 0)$, $f_y(0, 0)$ exist and function f is not continuous and differentiable at point $(0, 0)$.

(2) If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r\cos\theta$ then prove that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta.$$

(3) If $u = \frac{1}{r}$ and $r^2 = x^2 + y^2 + z^2$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

2 (a) If $u = \phi(H)$ is a function of a homogeneous function $H = f(x, y)$ of degree m whose partial derivatives of second order exist then prove that **8**

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = m\frac{F(u)}{F'(u)}$, $F'(u) \neq 0$

(ii) $$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = G(u)[G'(u) - 1]$$

where $H = f(x, y) = F(u) = \phi^{-1}(u)$

OR

2 (a) State and prove Taylor's theorem for functions of two variables. **8**

(b) Attempt any **two** : **10**

(1) If $u = \operatorname{cosec}^{-1}\left(\sqrt{\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}}\right)$ then prove that

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{-1}{12}\tan u$

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$

$$= \frac{\tan u}{144}\left(13 + \tan^2 u\right)$$

(2) Expand $f(x, y) = x^2 y + y^3$ in power of $x-2$ and $y-1$.

(3) Divide the number 24 into three parts so that the continued product of the first, square of the second and the cube of the third should be maximum.

3 (a) State and prove Rank-Nullity theorem. **8**

OR

(a) Let A be a nonempty subset of a vector space V. Show that [A] is the smallest subspace of V containing A. **8**

(b) Attempt any **two** : **10**

(1) Let

$U = \left[(a_1, a_2, a_3, a_4) / a_1 + a_2 = 1 = a_3 + a_4 \subset R^4 \right]$, then find dim U.

(2) A linear transformation $T : R^3 \to R^3$ is defined by

$T(x_1, x_2, x_3) = (x_1 + x_2 + x_3,\ x_2 + x_3,\ x_3)$, show that T is non singular and find its T^{-1}.

(3) Let $T : R^2 \to R^3$ be a linear transformation, $T(3, 2) = (2, -1, 3)$, $T(1, 1) = (5, 2, -1)$. Then obtain the formula for linear transformation.

4 (a) For a linear transformation $T : U \to V$, prove that range and rank of T are subspace of V, null space and nullity are subspace of U. **8**

OR

(a) Let W_1 and W_2 be two subspaces of a vector space $V(F)$. Then prove that

$W_1 + W_2 = [W_1 \cup W_2]$.

(b) Attempt any two : **8**

(1) Show that the set R^2 is a real vector space where vector multiplication is defined as follows.

$x = (a_1, b_1)$ and $y = (a_2, b_2)$ and

$ax = (\alpha a_1, 0)$.

(2) Verify Rank-Nullity theorem for a linear transformation $T : R^3 \to R^3$, $T(x_1, x_2, x_3)$

$= (-x_1 + x_2 + x_3, 2x_1 - x_3, x_1 + x_2 - 3x_3)$.

(3) Let the function $T : R^3 \to R^1$ defined by $T(\alpha, \beta, \gamma) = (\alpha^2 + \beta + \gamma)$. Then check whether T is linear transformation or not.