KT-5275-76-77 Seat No._____

B. Sc. (Sem. V) Examination

November / December – 2014

Mathematics

(1) CC-MATH-504-A - Mathematics

(2) CC-MAT-504-C - Operation Research - I (New Course)

Time : 3 Hours] [Total Marks : **70**

Instructions : (1) All questions are **compulsory**.
(2) Figures to the right indicate the marks of corresponding question.

(1) CC-MATH-504-A - Mathematics

1 (a) Define the following terms with illustration : **6**

(i) Equivalence relation

(ii) Lattice Homomorphism

(iii) Partially ordered set.

OR

(a) Let $\langle L, \leq \rangle$ be a lattice. Then for $a, b \in L$ prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$ **6**

(b) Attempt any **two** : **12**

(1) Define totally ordered set. In usual notation prove that $\langle I\!N, \leq \rangle$ is a TOSET but $\langle I\!N, D \rangle$ is not a TOSET.

(2) Draw the Hasse-Diagram of a poset $\langle S_6, D\rangle$ and $\langle S_6 \times S_6, D\rangle$.

(3) State and prove Absorption property for lattice.

2 (a) Define the following terms with illustration : **6**

(i) Modular lattice

(ii) Boolean Algebra

(iii) Complemented lattice

OR

(a) Let $\langle B, *, \oplus, 0, 1, '\rangle$ be a Boolean Algebra then prove that, for every $x_1, x_2 \in B$, **6**

(i) $x_1 \le x_2 \Rightarrow A(x_1) \subset A(x_2)$

(ii) $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$

(b) Attempt any **two** : **12**

(i) Show that $\langle S_{30}, D\rangle$ and $\langle \mathcal{P}(\{a,b,c\}) \subset\rangle$ are isomorphic lattices.

(ii) State and prove D'morgan's property for lattice.

(iii) Let $\langle \mathcal{P}(s), \cap, \cup, \phi, s, c\rangle$ and $\langle B, *, \oplus, 0, 1, '\rangle$ be two Boolean Algebras corresponding to the sets $S = \{a, b\}$ and $B = \{0, 1\}$. A mapping $f : \mathcal{P}(s) \to B$ is defined as follows : $\forall x \in \mathcal{P}(s)$, $f(x) = \begin{cases} 0 & ; \text{if } b \notin x \\ 1 & ; \text{if } b \in x \end{cases}$

show that f is a Boolean homomorphism but f is not a Boolean isomorphism.

3 (a) Define the following terms with illustration : **6**

(i) Boolean expression

(ii) Minterm

(iii) Max term

OR

(a) Express the Boolean function : **6**

$\alpha(x_1, x_2, x_3) = x_1 * (x_2 \oplus x_3')$ in a cube-array representation form.

(b) Attempt any **two** : **12**

(i) Let $\langle B, *, \oplus, 0, 1, ' \rangle$ be a Boolean Algebra. Show that a non-zero element a of B is an atom of B if and only if either $a * x = 0$ or $a * x = a; \forall x \in B$.

(ii) Obtain SOP canonical form of $\alpha(x_1, x_2, x_3) = x_1 \oplus x_3$

(iii) Minimize the Boolean expression $f(a, b, c, d) = \sum(0, 1, 2, 3, 13, 15)$ using the Karnaugh map representation.

4 Attempt any **two** : **8**

(i) Show that every chain is a lattice.

(ii) Show that every distributive lattice is modular.

(iii) In any Boolean Algebra B, show that $(a*b*c) \oplus (a*b) \oplus a = a$, for $a, b, c \in B$.

5 Attempt any **two** : 8

(i) For $P = \{2, 3, 6, 12, 24, 36\}$ show that $\langle P, D \rangle$ is a POSET but not a lattice.

(ii) In usual notations prove that $\langle S_n, D \rangle$ is a sublattice of $\langle IN, D \rangle$, $\forall n \in IN$.

(iii) Show that there is no Boolean Algebra of order 3.

(2) CC-MAT-504-C - Operation Research - I

(New Course)

1 (a) Prove that set of all feasible solution of the linear programming problem is a convex set. 10

(b) Solve the given Linear Programming Problem using SIMPLEX method. 10

Maximize $z = 8x_1 + 9x_2 + 5x_3$

Subject to condition

$x_1 + x_2 + 2x_3 \le 2$

$2x_1 + 3x_2 + 4x_3 \le 3$

$6x_1 + 6x_2 + 2x_3 \le 8$

$x_1, x_2, x_3 \ge 0$

OR

1 (a) Explain: Prove that an extreme point of the convex set of feasible solution is a basic feasible solution. 10

(b) Solve the given Linear Programming Problem using SIMPLEX method : **10**

Maximize $z = x_1 + x_2 + x_3$

Subject to condition

$2x_1 + x_2 - x_3 \leq 2$

$-2x_1 + x_2 - 5x_3 \geq -6$

$4x_1 + x_2 + x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$

2 (a) Define : **10**

(1) Convex set.

(2) Feasible Solution.

(3) Unbounded Solution

(4) Degenerate solution Slack Variable.

(5) Slack Variable.

(b) Solve the given Linear Programming Problem using Two Phase method : **10**

Minimize $z = -3x_1 + x_2$

Subject to condition

$x_2 \leq 4$

$x_1 + 3x_2 \leq 4$

$2x_1 + x_2 \geq 2$

$x_1, x_2 \geq 0$

OR

2 (a) Prove that the objective function of a linear programming problem attains its Optimal value at more than one extreme pointsthen for every convex combination of those points keep up the same optimal value. **10**

(b) Solve the given Linear Programming Problem using Big —M method : **10**

Minimize $z = 3x_1 + 2x_2$

Subject to condition

$2x_1 + x_2 \geq 2$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

3 (a) Prove that the dual of a dual is primal. **10**

(b) Solve the given Linear Programming Problem using Dual Simplex method : **10**

Minimize $z = 6x_1 + x_2$

Subject to condition

$2x_1 + x_2 \geq 3$

$x_1 - x_2 \geq 0$

$x_1, x_2 \geq 0$

OR

3 (a) If x_0 is a feasible solution to the primal problem **10**

Maximize $Z = cx$, such that $Ax = b; x \geq 0$ and w_0

is a feasible solution to the dual problem

Minimize $Z' = b'w$, such that $A'w = c'; w \geq 0$

then prove that $cx_0 \leq b'w_0$

(b) Solve the given Integer Programming Problem using Cutting Plane method. **10**

Maximize $z = 5x_1 + 5x_2$

Subject to condition

$2x_1 + 5x_2 \leq 16$

$6x_1 + 5x_2 = 30$

$x_1, x_2 \geq 0$

And are integers.

4 Attemt any **two** : **10**

(a) Solve the given Linear Programming Problem using Graphical method :

Maximize $z = 300x_1 + 400x_2$

Subject to condition

$5x_1 + 4x_2 \leq 200$

$3x_1 + 5x_2 \leq 150$

$5x_1 + 4x_2 \geq 100$

$8x_1 + 4x_2 \geq 80$

$x_1, x_2 \geq 0$

(b) For the given example prove that dual of the dual is primal :

Maximize $z = 5x_1 + 20x_2$

Subject to condition

$5x_1 + 2x_2 \leq 20$

$x_1 + 2x_2 \leq 8$

$x_1 + 6x_2 \leq 12$

$x_1, x_2 \geq 0$

(c) Solve the given Linear Programming Problem using appropriate method

Maximize $z = 6x_1 + 4x_2$

Subject to condition

$x_1 + x_2 \leq 5$

$x_2 \geq 8$

$x_1, x_2 \geq 0.$
