KT-5260 Seat No.________

B. Sc. (Sem. V) Examination

November / December – 2014

CC-MATH-502 : Analysis - I

Time : 3 Hours] [Total Marks : 70

Instructions : (1) All questions are **compulsory**, there are **five** questions.

(2) Figures to the right indicate marks of the corresponding question.

1 (a) State and prove Schwartz inequality for complex numbers. **6**

(b) If $\alpha \in R$, $\beta \in R$ and $\gamma = \{P \in Q / P = r + s, r \in \alpha, s \in \beta\}$ then show that $\gamma \in R$. **6**

(c) In usual notations prove that **6**

(i) $\alpha + 0^* = \alpha, \ \forall \alpha \in R$

(ii) $r^* + s^* = (r+s)^*$

OR

1 (a) Suppose S is an ordered set with least upper bound property. $B \subset S$, $B \neq \phi$, B is bounded below. Let L be the set of all lower bound of B, then prove that $\alpha = \sup L$ exists in S and $\alpha = \inf B$. **6**

(b) State and prove Archimedean property of R. **6**

(c) For complex numbers z and w. Prove that **6**

(i) $|z \cdot w| = |z||w|$

(ii) $|z + w| \leq |z| + |w|$

2 (a) Show that every k-cell is compact. **6**

(c) Suppose Y is a subspace of a metric space X. Prove that a subset E of Y is an open relative to Y if and only if $E = G \cap Y$, for some open set G of X. **6**

(c) Show that R is not compact. **6**

OR

2 (a) Show that a subset E of a metric space X is an open set if and only if its complement is closed. **6**

(b) Let E be an infinite subset of a compact set K, then prove that E has a limit point in K. **6**

(c) Let R be the set of all real numbers and $d : R \times R \to R$ be function such that **6**

$$d(x, y) = 1, \text{ if } x \neq y$$
$$= 0, \text{ if } x = y$$

show that d is metric on R.

3 (a) If $\{K_n\}$ is a sequence of compact subsets of metric space X such that **6**

$K_n \supset K_{n+1},\ n = 1, 2, 3, \ldots\ldots$ and if

$\lim\limits_{n\to\infty}$ diam $k_n = 0$ then prove that

$\bigcap\limits_{n=1}^{\infty} K_n$ consists of exactly one point.

(b) Define (i) Cauchy sequence (ii) Complete metric space. Show that R^K is complete metric space. **6**

(c) If $\{p_n\}$ and $\{q_n\}$ are Cauchy sequence in a metric space (X, d), then prove that sequence $\{d(p_n, q_n)\}$ converges in R. **6**

OR

3 (a) Suppose $a_1 \geq a_2 \geq a_3 \geq .... \geq 0$ then prove that **6**

$$\sum_{n=1}^{\infty} a_n \text{ converges if and only if}$$

$$\sum_{k=0}^{\infty} 2^k a_2 k = a_1 + 2a_2 + 4a_4 + ... \text{ converges.}$$

(b) Suppose (i) $\sum a_n$ converges absolutely **6**

(ii) $\sum_{n=0}^{\infty} a_n = A$, (iii) $\sum_{n=0}^{\infty} b_n = B$,

(iv) $a_n = \sum_{k=0}^{n} a_k b_{n-k}, n = 0, 1, 2, 3, ...$

then prove that $\sum a_n = AB$.

(c) Prove that the convergence of $\sum a_n$ implies **6**

the convergence of $\sum \frac{\sqrt{a_n}}{n}$ if $a_n \geq 0$.

4 Attempt any **two** : **8**

(1) Let A be a countable set and B_n be the set of all n-tuples $(a_1, a_2, a_3, ..., a_n)$, where $a_k \in A$, $k = 1, 2, 3, ..., n$ and $a_1, a_2, ..., a_n$ are need not be distinct. Prove that B_n is a countable set.

(2) Prove that $\sum \frac{1}{n^p}$ converges for $p > 1$ and diverges for $p \leq 1$.

(3) Let p be a limit point of subset E of a metric space X then prove that there exists a sequence $\{p_n\}$ in E such that $\lim_{n \to \infty} p_n = p$.

(4) Show that between any two real numbers, there is a rational numbers.

5 Attempt any **two** : **8**

(1) If $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2 + a_n}$, where $n = 1, 2, 3, ...$ then prove that sequence $\{a_n\}$ converges.

(2) Find the radius of convergence of the following series :

(i) $\sum_{n=1}^{\infty} \frac{3^n}{n+1} \cdot z^n$

(ii) $\sum_{n=0}^{\infty} \frac{n^3}{2^n (n+1)} z^n$

(3) Let A be a bounded above subset of R and $B = \{-x / x \in A\}$ then show that B is bounded below and inf. B = –sup A.

(4) Let $A = \left\{ \frac{1}{n} \middle/ n \in N \right\}$ then show that '0' is limit point of A.
