KT-5253 Seat No.________

B. Sc. (Sem. V) Examination

November / December – 2014

Mathematics : Paper - CC - MATH - 501

(Group Theory)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Figures to the right indicate the marks of the corresponding question.

1 (a) Define a group G, Show that the identity and inverse of a group is unique. **8**

OR

(a) State and prove Lagrange's theorem. **8**

(b) Attempt any **two** : **12**

(i) Show that the group of order 5 is always abelian.

(ii) Show that the set $G = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \Big/ a^2 + b^2 \right\}$ of 2×2 matrices is an abelian group under matrix multiplication.

(iii) If G is a group , $a^5 = e$, $aba^{-1} = b^2$ for $a, b \in G$ then o(b).

2 (a) Define a normal subgroup. Prove that a subgroup H of a group G is normal iff $Ha.Hb = Hab, \forall a,b \in G$. 8

(b) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 1 & 2 & 6 & 10 & 4 & 9 & 8 & 5 & 7 \end{pmatrix} \in S_{10}$ 6

then express: (1) f as a product of a disjoint cycles (2) f as a product of transposition (3) find $o\left(f^{-1}\right)$.

(c) Show that the group $(G = \{0, 1, 2, 3\}\ +_4)$ is isomorphic to the group $\left(\bar{G} = \left(\{1, 2, 3, 4\}, x_5\right)\right.$. 6

OR

2 (a) Prove that any two disjoint cycles in S_n are commutative. 8

(b) Define congruence modulo relation. Solve the congruence $95x \equiv 165 \pmod{15}$. 6

(c) If $\langle R^+, \times \rangle$ and (R, +) be two groups then show that mapping $f : R^+ \to R$ defined by $f(x) = \log x,\ x \in R^+$ is an isomorphism. 6

3 (a) State and prove Fundamental Theorem of Homomorphism. 8

OR

(a) State and prove Cayley's Theorem. 8

(b) Attempt any **two** : 12

(1) If $\phi : (G, *) \to (G', 0)$ is a homomorphism then prove that ϕ is one-one iff $K_\phi = \{e\}$.

(2) Find all the subgroups of $G = \langle a \rangle$ cyclic group of order 12 .Also find order of each element of G and find other generators of G.

(3) Show that two cyclic groups of same order are isomorphic .

4 Attempt any **two** : **10**

(a) If a is an element of order n and p is a prime to n then a^p is also of order n.

(b) Determine which of the following mapping is homomorphism and if so obtain its kernel

(i) $G_1 = (z, +); G_2 = (R, +)$ and $\phi : G_1 \to G_2$, with $\phi(x) = x, \forall x \in G_1$.

(ii) $G_1 = (R, +); G_2 = (R - \{0\}, \times)$ and $\phi : G_1 \to G_2$ with $\phi(x) = 2^x, \forall x \in G_1$.

(c) Let (G = {1, 2, 3, 4} × 5) is a group and sub group H {1, 4} then find the distinct right cosets of H in G .
