KN-5862 Seat No.________

B. Sc. (Sem. III) Examination

November / December - 2014

MATH-301 : Mathematics

Time : 3 Hours] [Total Marks : 70

1 (a) State and prove Schasrtz's theorem. 8

OR

(a) Suppose $Z = f(x, y)$ is defined on a nonempty set $E \subset R^2$ and f_x, f_y exist and continuous at (x, y) then prove that f is differentiable at $(x, y) \in E$. 8

(b) Attempt any **three** : 12

(1) If $f(x, y) = \left.\begin{matrix} \dfrac{x^2 - y^2}{x^2 + y^2}, x \neq 0, \ y \neq 0 \\ = 5, \qquad\qquad x = y = 0 \end{matrix}\right\}$ then by definition of limit prove that $\lim\limits_{(x, y) \to (0, 0)} f(x, y)$ does not exist.

(2) If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$ then prove that $\left(\dfrac{\partial}{\partial x} + \dfrac{\partial}{\partial y} + \dfrac{\partial}{\partial z}\right)^2 u = \dfrac{-9}{(x + y + z)^2}$

(3) If $F(x, y, u, v) = x^3 + y^3 + u^3 + 2v^3 - 5 = 0$

$G(x, y, u, v) = 2x^3 - y^3 + 3u^3 - v^3 - 7 = 0$

then find $\frac{\partial^2 u}{\partial x^2}$.

(4) If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$ then prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = u$$

2 (a) State and prove Euler's theorem. **8**

OR

(a) State and prove Taylor's theorem. **8**

(b) Attempt any **three** : **12**

(1) Verify Euler's theorem

$$f(x) = x^2 - \tan^{-1}\frac{y}{x} - y^2 \cdot \tan^{-1}\frac{x}{y}$$

(2) Using Maclaurin's theorem prove that

$$2^{ax} \cdot \cos by = 1 + ax + \frac{1}{2!}\left(a^2x^2 - b^2y^2\right) + \ldots\ldots\ldots\ldots$$

(3) Find the extreme value of

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20$$

(4) Show that all triangles having given perimeter, the largest area is in an equilateral triangle.

3 (a) State and prove Rank-Nullity theorem. 8

OR

(a) For a Linear Transformation $T : U \to V$ prove that the range $T, R(T)$ and Null space of $T, N(T)$ are subspaces of V and U respectively. 8

(b) Attempt any **three** : 12

(1) Find the co-ordinates of the polynomial $2x^2 + 7x + 3$ relative to the ordered basis $\left\{\left(1-x, 1+x, 1-z^2\right)\right\}$ of vector space $P_2(R)$.

(2) Define a basis, prove that the set $B = \{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$ is a basis of vector space R^3.

(3) Which of the following subsets of a vector space R^3 are subspaces ?

(i) $w_1 = \{(a, 4a, 3a + 2/a \in R)\}$

(ii) $w_2 = \{(a-b, a+b, 2a-3b/a, b \in R)\}$

(4) Find $R(T), N(T), r(T), n(T)$ for linear transformation $T : R^2 \to R^3$,

$T(x_1, x_2) = (x_1 + x_2, x_1 - x_2, x_2)$

4 Attempt any **five** : 10

(1) Show that $w=\{(t, 2t, -3t)/t \in R\}$ is a subspace of R^3.

(2) Find a linear transformation $T: R^2 \to R^3$ such that $T(1,1)=(2,0,1)$, $T(2,-1)-(1,-1,1)$

(3) If $f(x, y)=\frac{xy^3}{x^2+y^6}$, $x \neq 0$, $y \neq 0$ and $f(0,0)=0$. Discuss continuity of f at (0, 0).

(4) Find $\frac{d^2 y}{dx^2}$ for $x^2+y^2=1$.

(5) If $u=\tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, $x+y \neq 0$ then prove that

$$x \cdot \frac{\partial u}{\partial x}+y \cdot \frac{\partial u}{\partial y}=\sin 2u.$$

(6) Find the equation of tangent plane of $2xz^2-3xy-4x=7$ at $A(1,-1,2)$.

(7) If $f(x, y)=\frac{x^2+y^2}{x+y}$, $x \neq 0$, $y \neq 0$ then prove that

$$\left[1-\frac{\partial f}{\partial x}-\frac{\partial f}{\partial y}\right]-\frac{1}{4}\left(\frac{\partial f}{\partial x}-\frac{\partial f}{\partial y}\right)^2=0$$
