P.S.SCIENCE & H.D.PATEL ARTS COLLEGE, KADI
INTERNAL EXAMINATION

B.Sc. Sem - V [Marks 40

10/09/2017 Mathematics:CC-MATH- 502 [1.45 to 3.45

1. (a) State and prove Schwart'z inequality.

   (b) If $\alpha \in R$, $\beta \in R$ and $\gamma = \{p \in Q / p = r+s, r \in \alpha, s \in \beta\}$ then show that $\gamma \in R$.

   (c) Suppose $\overline{a}, \overline{b} \in R^k$ find $\overline{c} \in R^k$ and $r > 0$ such that $|\overline{x} - \overline{a}| = 2|\overline{x} - \overline{b}|$ iff $|\overline{x} - \overline{c}| = r$.

Or

1. (a) Suppose S is an ordered set with the least upper bound property. Let $B \subset S, B \neq \phi$ and B is bounded below. Let L be the set of all lower bound of B then $\alpha = \sup L$ exists in S and $\alpha = \inf B$.

   (b) Prove that R has least upper bound proprety.

   (c) For $\overline{x}, \overline{y} \in R^k$ prove that $|\overline{x}+\overline{y}|^2 + |\overline{x}-\overline{y}|^2 = 2|\overline{x}|^2 + 2|\overline{y}|^2$.

   Interpreter this geometrically as a statement above parallelogram.

2. (a) Let $\{E_n\}_{n=1}^{\infty}$ be the collection of countable sets then show that $\bigcup_{n=1}^{\infty} E_n$ is also a countable set.

   (b) Prove that every compact subset of metric space is closed.

   (c) Suppose $\overline{a}, \overline{b} \in R^k$ find $\overline{c} \in R^k$ and $r > 0$ such that $|\overline{x} - \overline{a}| = 2|\overline{x} - \overline{b}|$ iff $|\overline{x} - \overline{c}| = r$.

Or

2. (a) Show that the set F is closed iff its complement is an open set.

   (b) Show that a subset E of the real line R^1 is connected if and only if it satisfies the following property.
   Is $x, y \in E$ and $x < z < y$ then $z \in E$.

   (c) Let R be the set of all real number and $d : R \times R \to R$
   $(i) d(x,y) = 1, x \neq y$ (ii) $d(x,y) = 0$ if $x = y$
   then show that d is metric on R.

3. (a) If $E \subset X$ and P is a limit point of E then there exists a sequence $\{P_n\}$ in E such that $P = \lim_{n \to \infty} p_n$

(b) If $\{K_n\}$ is a sequence of compact sets in a metric space X such that $K_n \supset K_{n+1}$ $(n = 1, 2, 3, \ldots)$ and if $\lim_{n\to\infty} diam\ K_n = 0$ then $\bigcap K_n$ consists of exactly one point.

(C) Prove that $\lim_{n \to \infty} \sqrt[n]{n} = 1$

Or

3. (a) Suppose (i) the partial sum A_n of $\sum a_n$ from a bounded

Sequence (ii) $b_0 \geq b_1 \geq b_2 \geq \cdots . \geq 0$

(iii) $\lim_{n\to\infty} b_n = 0$

Then prove that $\sum a_n b_n$ converges.

(b) Prove that the series $\sum \frac{1}{n^p}$ is converges, if $p > 1$ and divergence if $p \leq 1$.

(c) if $S_1 = \sqrt{2}$ and $S_{n+1} = \sqrt{2} + S_n$ then prove that $\{S_n\}$ converges and $S_n < 2$.

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