Pramukh Swami Science & H D Patel Arts College, Kadi
Internal Examination, Oct-2017,
B.Sc. Semester- V (Mathematics)
CC MATH-501 Group theory

Date: 17/09/2017

Time:2 hrs. **Total Marks: 40**

1. (a) Define a Group. Show that, **[06]**

(i) Identity is unique in any group

(ii) Inverse of each element is unique

OR

(a) Show that, the set of all n^{th} roots of unity forms a finite abelian group with order n under usual multiplication.

(b) Attempt any three of the following: [09]

1) Show that, the group G is abelian group if $(ab)^k = a^k b^k$ a,b∈G, for any three consecutive integers starts from k.

2) Let G be a finite group with order n and a ∈G then show that, o(a)$\leq G$

3) Let H be a non-empty subset of a group G then H is a subgroup of G iff $a, b \in G \Rightarrow ab^{-1} \in G$.

4) Give the proper example of

(i) non - abelian group with order 6

(ii) abelian group with order 8

(iii) infinite non abelian group

2. (a) A subgroup H is Normal subgroup of a group G

iff $xHx^{-1} \subseteq H, \quad \forall x \in G.$ **[06]**

OR

(a) Define a quotient group of Normal subgroup N in group G. and show that, the product of two right cosets Na & Nb for a,bG is again a right coset of N i.e. (Na)(Nb)=Nab

(b) Attempt any two of the following: [06]

1) Define the quaternion group and give its example.
2) Show that, the centre Z of a group G is a normal subgroup of G.
3) Prove that, any two finite cyclic groups with same order are always isomorphic. Hence show that, $G \cong (Z_n, +_n)$ for any finite cyclic group G with order n .
4) If K is subgroup of group G and H is normal subgroup of G then show that, $H \cap K$ is a normal subgroup of K

3. **(a)** State and prove Langrange's theorem, hence define the index of subgroup in group. **[07]**

OR

Prove that, no permutation can be both even as well as odd together in S_n

(b) Attempt any two of the following: [06]

1) Find the remainder on dividing 29^{2017} by 13. (using Euler's theorem)
2) check whether given permutation is even or odd :

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 8 & 6 & 11 & 10 & 9 & 12 & 5 & 3 & 7 & 2 & 1 & 4 \end{pmatrix} \in S_{12}$$

3) Define alternating subgroup of S_n and Show that,

$$o(A_n) = o(B_n) = \frac{n!}{2}$$

BEST OF LUCK