P.S.SCIENCE & H.D.PATEL ARTS COLLEGE, KADI

INTERNAL EXAMINATION

B.Sc. Sem -I [Marks 40

10/09/2017 Mathematics 111 [1.45 to 3.45

1. [A] State and Prove Leibnitz's theorem.

   [B] Attempt any two :

   1. If $y = cos^{-1}(x), x \in (-1,1)$ then prove that, $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$
   2. Prove that, $\frac{4}{3} < \frac{log\,3}{cot^{-1}2} < 4$. where$x \in [1,3]$
   3. Find the co-efficient of x^4 in the expansion of $log(cosx)$

2. [A] Find the reduction formulae of $\int sin^n x\,dx, n \in N$.

   [B] Attempt any two :

   1. Evaluate : $\int_0^1 x^4(2-x^2)^{\frac{3}{2}}\,dx$.
   2. Find the value of
      $$\frac{1}{n}\left[sec^2\left(\frac{\pi}{4n}\right) + sec^2\left(\frac{2\pi}{4n}\right) + \cdots + sec^2\left(\frac{n\pi}{4n}\right) + \cdots\right]$$
   3. Find the surface area of cone whose base radius is r and height is h

3. [A] Obtain the polar equation of line passing through $A(r_1,\theta_1)$ and $B(r_2,\theta_2)$.

   [B] Attempt any two :

   1. Find the polar equation of line passing through $A(2,\frac{\pi}{6})$ and $B(\sqrt{3},\frac{\pi}{3})$
   2. Prove that, the length of the focal chord of a parabola which makes an angle $\frac{\pi}{6}$ with its axis is four times its latus rectum.
   3. Find curl and divergence of vector $\vec{F} = (xyz, 3x^2y, xz^2 - y^2z)$.

4. [A] Obtain the equation of tangent plane to the sphere $x^2 + y^2 + z^2 = a^2$ at point $A(x_1,y_1,z_1)$

[B] Attempt any two :

1. Find the intersection points of st. line $\frac{x+3}{4} = \frac{y+4}{3} = \frac{z-8}{-5}$ and the sphere $x^2 + y^2 + z^2 + 2x - 10y = 23$
2. Prove that, the equation $xy + yz + xz = 0$ represents right circular cone. Also find semi vertical angle and equation of axis.
3. Find the equation of the cylinder whose generator line is parallel to Z-axis and guiding curves are $x^2 + y^2 + z^2 = 8, x + 2y + 2z = 6.$